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Journal
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and
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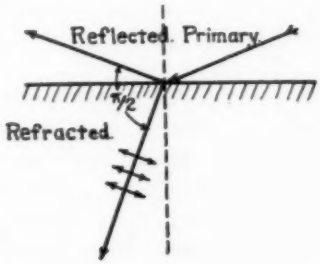
Vol. 7 JULY, 1923 Number 7

NOTE ON BREWSTER'S LAW

By A. Sommerfeld

Brewster's Law concerning the angle of polarization seems to be a rather peculiar and artificial expression of the optical facts. We will show, that it is indeed the most natural one from the standpoint of electronic radiation. We do not deal here with the modifications, perhaps necessary from the views of quantic theory, but will rest on classical considerations.



If a beam of light falls on the plane of an optically denser medium then, according to the theory of dispersion, the electrons in the second medium oscillate in the direction of the electric force of the refracted beam, that is perpendicularly to the direction of the refracted beam. They produce electromagnetic radiation in the same way as we used to calculate scattered radiation. This radiation has to create the reflected beam, coming from the second medium, the second medium itself being excited by the primary beam.

Now the most fundamental feature of classical radiation is its transversal character. That means that we have no radiation in the direction of the electronic motion. As this motion is perpendicular to the refracted beam, we have no radiation and no reflected beam, if this beam would be perpendicular to the refracted ray. That is Brewster's Law.

We see immediately from the diagram, that the foregoing remarks hold only in the case the electric force falls in the plane of incidence (plane of the diagram), that means in the case, the plane of polarization is perpendicular to the plane of incidence. The short arrows in the diagram indicate the direction of electronic oscillations in this case; they are unable to produce any reflected beam.

In the opposite case the oscillations of the electrons are perpendicular to the plane of the diagram. Here they radiate the maximum amount in the direction of reflection. Therefore they are able to produce by interference the reflected beam, which is, in consequence, polarized totally in the plane of incidence.

Concluding, we wish to emphasize that the phenomenon of reflection (as well as that of refraction or diffraction) is identical with that of scattering, in the classical treatment of those phenomena.

UNIVERSITY OF MUNICH,
MUNICH, GERMANY.

REGULARITIES IN THE SCREENING CONSTANTS OF RÖNTGEN SPECTRA

By A. Sommerfeld

It is a well known fact that the fine structure theory of the hydrogen atom accounts, with a remarkable degree of accuracy, for the doublet difference of the L absorption limits L_1, L_2. This same difference in frequency is brought out anew in the $(\alpha\ \alpha')$ doublet of the K emission series as well as in the $(\alpha'\ \beta)$, $(\gamma'\ \delta)$, $(\epsilon\ \eta)$, $(\zeta\ \theta)$ etc., etc. doublets of the L emission series. A single empirical constant, the screening constant $s = 3.50$, together with the universal constant $\alpha = 4\pi e^2/ch$, suffices to account for this difference for all elements within the limits of error of the best experimental determinations. The formula representing this difference is composed of these constants and nothing else but integral numbers. According to the computations of E. Hjalmar[1] this same formula with properly selected values of the screening constants gives the doublet differences of the $M_1\ M_2$ and $M_3\ M_4$ absorption limits which are equal respectively to the $(\alpha\ \alpha')$ and $(\phi\ \phi')$ of the L emission series. Thereupon G. Wentzel in a joint article[2] with the author studied the doublet differences $N_1\ N_2$, $N_3\ N_4$, $N_5\ N_6$ of the various N levels and deduced the screening constants which correspond to them. These constants are naturally much more inaccurate than those mentioned above, the corresponding energy differences being very small.

We will call all these doublets "relativity doublets" as they are represented by means of a formula which is derived from the special relativity theory, and as they most likely owe their origin to that part of the electron path which passes near the nucleus where relativity demands a large increase in the mass of the electron.

But there exists still an entirely different class of doublets which we will call "screening doublets," namely, the doublets $L_2\ L_3$; $M_2\ M_3$, $M_4\ M_5$; $N_2\ N_3$, $N_4\ N_5$, $N_6\ N_7$. The nature of these was first recognized by G. Hertz. The expressions "relativity and screening doublets" were proposed by D. Coster; the expressions "regular and irregular doublets" originally used by G. Wentzel and the author are inapt.

The relativity doublets are characterized by a screening constant, s; the screening doublets, on the other hand, by Δs, the difference be-

[1] ZS. f. Physik *3*, 262; 1920.
[2] ZS. f. Physik *7*, 86; 1921.

tween two screening constants. In his "habilitation" thesis,[3] G. Wentzel finds that a remarkable integral relation exists between these Δs values. A glance at the values of the screening constants s of the relativity doublets shows us that a similar relation appears here also. In both cases we have to deal with an empirical formulation; for the present we are unable to say what these regularities signify in the atomic model.

1. REGULARITIES IN THE SCREENING CONSTANTS OF THE RELATIVITY DOUBLETS

In Table 1 we group together with a brace two energy levels which form a relativity doublet. By this we mean that the difference in energy divided by $R\,h$ is given by the following formula.[4]

$$(1)\quad \frac{\Delta\nu}{R}=\frac{\alpha^2(Z-s)^4}{n^4}\left(a_1+a_2\frac{\alpha^2(Z-s)^2}{n^2}+a_3\frac{\alpha^4(Z-s)^4}{n^4}+\ \ldots\ \right)$$

Here, n is Bohr's total quantum number. (In my previous notation this was the "quantum sum" $n+n'$.) It equals 2 for the L doublet; 3 for the M doublets and 4 for the N doublets. The numbers a_1, a_2, a_3 are rational numbers which are given in my book.[5] They are composed of n and the azimuthal quantum numbers of the two energy levels concerned; e.g., for the L doublet they are: $a_1=1$, $a_2=\frac{5}{2}$, $a_3=\frac{53}{8}$.

TABLE 1

K						
	L_1 L_2			L_3		
	3.50					
M_1		M_2	M_3		M_4	M_5
	$13.16=3\times4.39$		$8.25=2\times4.12$			
N_1	N_2	N_3	N_4	N_5	N_6	N_7
$33.9=8\times4.2$		$25.7=6\times4.3$		$16.3=4\times4.1$		

Beneath each doublet is inserted the screening constant s which when substituted in formula (1) makes this represent the observed doublet difference within the experimental error. As we remarked in the

[3] München, 1922.

[4] How many terms are to be used in this formula is discussed by A. Sommerfeld and W. Heisenberg in ZS.f. Physik *10*, p. 393; 1922. See also F. Green, Phys. Rev.; 1923 (to appear soon).

[5] Atombau und Spektrallinien. 3rd Ed. Braunschweig, 1922. pp. 607, 617, 619; 1922.

introduction, the s values for the N doublets are not as exact as those for the L and M doublets. At the same time we must note that s in formula (1) occurs only in the factor $(Z-s)$, that is, it is to be subtracted from the large atomic number Z. For this reason the evaluation of any of the s values is made somewhat difficult.

We now see that *the various s values of the M and N doublets are integral multiples of a common basic unit which lies between 4.1 and 4.4.* The sequence of the integral multipliers is also characteristic:

$$\underbrace{M_4\ M_3}_{2} \qquad \underbrace{M_2\ M_1}_{3} \qquad \underbrace{N_6\ N_5}_{4} \qquad \underbrace{N_4\ N_3}_{6} \qquad \underbrace{N_2\ N_1}_{8}$$

Their growth corresponds to a passage from low to high energy levels. For example, the levels $M_4\ M_3$ are beneath the levels $M_2\ M_1$; the former are nearer to the nucleus than the latter. We, therefore, readily see why the doublet $M_4\ M_3$ possesses a smaller screening constant, 2×4.12, than the doublet $M_2\ M_1$, which has 3×4.39. The energy levels of the latter are farther from the nucleus and are consequently more strongly screened than the energy levels of $M_4\ M_3$.

It is true however, that the L doublet, which is the most accurately known of all, does not fit this rule. Its screening constant $s=3.50$ is certainly different from the basic unit of the remaining relativity doublets. Also the small discrepancies in the basic unit of the two M doublets 4.39 and 4.12 may be real and may lie outside the limits of experimental error.

2. REGULARITIES IN THE DIFFERENCE OF THE SCREENING CONSTANTS OF THE SCREENING DOUBLETS

In Table 2 a brace joins two energy levels which form a screening doublet. By this we mean that two such energy levels are given by formulas[6] of the following type.

$$(2)\quad \begin{cases} \dfrac{\nu'}{R}=\dfrac{(Z-s)^2}{n^2}+\dfrac{\alpha^2(Z-\sigma')^4}{n^4}\left(a_1+a_2\dfrac{\alpha^2(Z-\sigma')^2}{n^2}+\ \ldots\ \right) \\ \dfrac{\nu''}{R}=\dfrac{(Z-s+\Delta s)^2}{n^2}+\dfrac{\alpha^2(Z-\sigma'')^2}{n^4}\left(a_1+a_2\dfrac{\alpha^2(Z-\sigma'')^2}{n^2}+\ \ldots\ \right) \end{cases}$$

These two formulas are distinguished in the first place by the first term. This is the principal term and has the same character as Moseley's representation of the lines $K\ \alpha$, $L\ \alpha$. . . . While ν' contains the screening constant, s, ν'' is composed of $s-\Delta s$; we can therefore foresee

[6] Compare this with the previous citation: ZS. f. Physik 7, p. 86; 1921.

that ν'' belongs to the deeper energy level i.e., to the higher absorption limit. In the case of the screening doublet $L_2\ L_3$, for example, ν' would go with L_2 and ν'' with L_3.

The two formulas differ in the second place by the screening constants in the relativity correction.[7] In contrast with the usage in the first paragraph of section 1, in which we designated these constants by s, we will here call them σ' and σ''. They are known if the energy level in question occurs in a relativity doublet as well as in the screening doublet under discussion. In the case of the screening doublet $L_2\ L_3$, for example, $\sigma' = 3.50$ while σ'' is at present still unknown.

TABLE 2

K						
L_1	L_2	L_3				
	$1.21 = 2 \times 0.60$					
M_1	M_2	M_3	M_4	M_5		
	$3.39 = 6 \times 0.57$		$1.72 = 3 \times 0.57$			
N_1	N_2	N_3	N_4	N_5	N_6	N_7
	$7.0 = 12 \times 0.6$		$4.7 = 8 \times 0.59$		$2.3 = 4 \times 0.57$	

Beneath each doublet is given in Table 2 the difference in the screening constants Δs, as it was defined in formula (2). We now can see that *the various values of Δs are integral multiples of one and the same basic unit 0.57*. (As we stated in the introduction G. Wentzel was the first to call attention to this rule.) This is strictly true, particularly in the case of the M doublets. The small discrepancies in the N levels undoubtedly lie within the limits of error. Only in the case of the L doublet the discrepancy (0.60 instead of 0.57) seems to be real; so that here, as in the case of the relativity doublets, the regularity holds only approximately.

If we now arrange the screening doublets in the order of their energy levels going from low to high levels, the series of integral multipliers again becomes very characteristic.

$L_3\ L_2$	$M_5\ M_4$	$M_3\ M_2$	$N_7\ N_6$	$N_5\ N_4$	$N_3\ N_2$
2	3	6	4	8	12

In a general sort of a way, this series parallels the increasing screening caused by the intermediate shells.

[7] The fact that the two screening constants s and σ are different seems to indicate that they owe their origin to different parts of the electron path: σ to the parts nearest the nucleus, s to the average path.

3. HYPOTHETICAL CONSEQUENCES

In Table 1 the screening constants of the relativity factors remained undetermined for the present in the cases of the lowest levels of each shell, i.e., in N_7, M_5, L_3 and K, since these levels do not occur in any relativity doublet. It is quite likely possible, however, to extrapolate the rule of Table 1 and to expect the following values for the screening constants:

N_7	M_5
2×4.1	1×4.1

A corresponding extrapolation for L_3 and K is evidently impossible. The test of this expectation is intimately connected with the general problem of finding the terms in the domain of X-rays. Evidently, the final goal of X-ray spectroscopy is the determination of the various K, L, M, N, . . . terms. This goal seems to be much closer than the corresponding one in the visible region. In fact our formula (2) states that only two empirical constants, there designated by s and σ, are necessary to determine each term; and our Tables 1 and 2 state that the assemblage of these pairs of constants can be reduced strictly or approximately to two basic units. Besides these two basic units there would then remain in the representation of the terms only the screening constants s in the principal terms of the highest levels of each shell, namely, K, L_1, M_1, N_1. . . . We must also reiterate in this summary the uncertainty in the relativity terms of the K and L_3 levels. In this way the problem of X-ray terms promises a very simple solution.

How the screening constants and their regularities are to be explained theoretically is a question for a later time. We repeat that the regularities discussed herein are, for the present, of an empirical nature.

After this had been written, the author learned that Bohr and Coster studied in Zeitschrift für Physik *12*, p. 342; 1923 the terms and the screening constants of X-ray spectra, both in their empirical and theoretical aspects. They pointed out that the screening constant β (our s in the chief term of formula 2) is expected from the theory not to be constant but to change with the atomic number from one period to the other in a regular way. The numerical values they give show this for the levels K_1, L_1, M_1, N_1. These changes belong to that part of the screening constant, which Bohr and Coster call the "outer screening effect β." In our differences, Δs, this part evidently is eliminated. The regularities here stated in paragraph 2 on the other hand belong to that part of the screening constant, which Bohr

and Coster attribute to the "inner screening effect a." The values of the screening constants in the relativity doublets are not studied by Bohr and Coster. In this way the regularities pointed out in the present paper seem to complete the statements given by Bohr and Coster.

MUNICH, GERMANY.

X-ray Spectra and the Periodic System of the Elements.—This important paper by Bohr and Coster is a correlation of the system of X-ray spectra with the theory of atomic structure. Valuable material is presented in tables and diagrams as follows. Table 1 and Fig. 1 show the Bohr scheme of electron orbits appearing in successive elements in the periodic table. Fig. 2 is an energy diagram of X-ray levels showing the origin of all emission lines. Table 2 gives ν/R values for all levels which are observed or can be computed from emission spectra and Table 3 gives corresponding $\sqrt{\nu/R}$ values. Figs. 3 and 4 show $\sqrt{\nu/R}$ plotted against atomic number N.

An approximate formula for the value ν/R of any energy level is derived from theoretical considerations

$$\nu/R=\frac{(N-a-n^2\beta)^2}{N^2} \qquad (1)$$

where n is the total quantum number, a is a constant for corresponding levels in different elements and β is a term approximately equal to the number of the period in the periodic table i.e. 1 for helium, 2 for neon, etc. Experimental evidence for the existence of this β term is given.

Some serious difficulties in the complete explanation of X-ray fine structure are pointed out but except for this the diagrams of Figs. 3 and 4 give striking confirmation of the theory. The 3_3 levels begin near the iron group, 4_3 with the palladium group, and 5_1, 5_2 and 4_4 levels appear successively near the rare earth group. Another consequence of the theory is that the Moseley diagram lines are not smooth curves but have a decreased slope in long periods. This is expressed in the postulated value of β in eq(1). The effect becomes conspicuous only in levels near the atom surface. Coster's accurate and extensive X-ray data show this phenomenon conclusively, particularly in the N and O series of the rare earth group. In fact in the diagrams (Figs. 3 and 4) is outlined nearly the whole system of atom building. The order of appearance of new types of orbits and the resulting effect on previously existing orbits is clearly indicated. [N. Bohr and D. Coster ZS. f. Physik *12*, p. 342; 1923].

F. L. MOHLER

THE MODEL OF THE NEUTRAL HELIUM ATOM

By A. Sommerfeld

All attempts made hitherto to solve the problem of the neutral helium atom have proved to be unsuccessful. The original Bohr model,[1] with the two electrons moving in the same direction on a single circle, at opposite extremities of a diameter, gives an ionization potential of 28.75 volts; moreover, it is paramagnetic, possessing a magnetic moment of $2 \cdot \frac{e}{m} \cdot \frac{h}{4\pi}$ (two Bohr magnetons). The model proposed by Kemble[2] and supported by Bohr, with its two approximately circular crossed orbits in planes making an angle of 60°, gives an ionization potential of about 20 volts, according to Van Vleck.[3] This model also is paramagnetic, having one magneton. The correct ionization potential has been shown by Lyman[4] to be 24.5 volts.

Bohr takes the view that the magnetic moment of the atom can not be calculated from its angular momentum as was presupposed in the foregoing statements; so far as I can see this view has no basis of observed fact. Therefore we must conclude from the diamagnetic behavior of the neutral helium atom that the angular momenta of the two electrons always have a vanishing resultant; this can be true only if the planes of the two orbits coincide, so we assume a coplanar model, with the electrons moving in opposite senses about the nucleus. The quantum conditions are derived by the following reasoning in connection with the hydrogen atom.

1. Quantum Conditions for the Hydrogen Atom, Considering the Motion of the Nucleus

So far as I know, the only case in which we are certain of the quantum conditions for a system having more than one moving body, is that of the hydrogen atom. I pointed out in 1916,[5] that if we set the sum of the azimuthal phase integrals for both the electron and the moving nucleus equal to an integral multiple of h, and do the same for the radial phase integrals, we obtain the correct Rydberg constant for hydrogen, given by Bohr for the case of circular orbits. Let r, φ, R, Φ, be the polar coordinates of the electron and the nucleus, respectively,

[1] Phil. Mag., *26*, pp. 1 and 476; 1913.
[2] Phil. Mag. *42*, p. 123; 1921.
[3] Phil. Mag. *44*, p. 842; 1922.
[4] Science, *16*, p. 167; 1922.
[5] Ann. der Physik, *51*, p. 39; 1916.

referred to their common center of gravity; ρ, the distance between them; and m, M, their respective masses. Then

$$\rho = r + R, \; r = \frac{M}{M+m}\rho, \; R = \frac{m}{M+m}\rho;$$

(1)

$$\varphi - \Phi = \pi, \qquad \dot{\varphi} = \dot{\Phi}, \qquad d\varphi = d\Phi.$$

The momenta or impulses may be denoted by

(2) $$p_r = m\dot{r}, \; p_R = M\dot{R}, \; p_\varphi = m r^2 \dot{\varphi}, \; p_\Phi = M R^2 \dot{\Phi},$$

and we assert that the correct quantum conditions are

(3) $$\left.\begin{cases} \int p_r dr + \int p_R \, dR = n'h, \\ \int p_\varphi d\varphi + \int p_\Phi d\Phi = kh, \end{cases}\right\} \; n', \; k = 1, \; 2, \; 3, \ldots\ldots\ldots .$$

The integers n' and k are the radial and azimuthal quantum numbers respectively. The integrals are extended over one complete period of the system. Equations (3) are proved correct by deriving from them the usual quantum conditions for the system, as seen by an observer who is fixed with respect to the nucleus. Using (1) and (2) in (3) we get

(4) $$\begin{cases} \int p_r dr = m \int \dot{r} \, dr = m \, \dfrac{M^2}{(M+m)^2} \int \dot{\rho} \, d\rho \\ \int p_R \, dR = M \int \dot{R} dR = M \, \dfrac{m^2}{(M+m)^2} \int \dot{\rho} \, d\rho \end{cases}$$

Now if we put for the relative motion,

$$\mu = \frac{m M}{M+m}, \quad p_\rho = \mu \dot{\rho},$$

equations (4) become

(5) $$\int p_r dr = \frac{M}{M+m} \int p_\rho d\rho, \; \int p_R dR = \frac{m}{M+m} \int p_\rho d\rho.$$

Similarly, from the 2d equation of (3),

(6) $$\begin{cases} \int p_\varphi d\varphi = m \int r^2 \dot{\varphi} d\varphi = m \dfrac{M^2}{(M+m)^2} \int \rho^2 \dot{\varphi} d\varphi, \\ \int p_\Phi d\Phi = M \int R^2 \dot{\Phi} d\Phi = M \dfrac{m^2}{(M+m)^2} \int \rho^2 \dot{\varphi} d\varphi. \end{cases}$$

If we introduce the whole angular momentum of the system, viz.,

$$p = \mu \rho^2 \varphi = p_\varphi + p_\Phi = \text{constant},$$

we see from (6) that

(7) $$\int p_\varphi d\varphi = \frac{M}{M+m} \int p d\varphi, \; \int p_\Phi d\Phi = \frac{m}{M+m} \int p d\varphi.$$

Putting (5) and (7) into (3), we conclude

$$\int p_\rho d\rho = n'h, \quad \int p\, d\varphi = kh. \tag{8}$$

These are the right quantum conditions for the relative motion of the electron with respect to the nucleus, for they yield the correct value of the Rydberg constant,

$$R_H = \frac{2\pi^2\mu e^4}{h^3} = \frac{2\pi^2 m e^4}{h^3\left(1+\frac{m}{M}\right)} = \frac{R_\infty}{1+\frac{m}{M}}. \tag{9}$$

Using (8), equations (5) and (7) may be written

$$\begin{cases} \int p_r dr = \dfrac{M}{M+m}\, n'h, \quad \int p_R dR = \dfrac{m}{M+m}\, n'h, \\ \int p_\varphi d\varphi = \dfrac{M}{M+m}\, kh, \quad \int p_\Phi d\Phi = \dfrac{m}{M+m}\, kh. \end{cases} \tag{10}$$

We state therefore: The single phase integrals in the problem of two bodies moving about a fixed center of force are multiples of a fraction of h; only the sums of corresponding phase integrals for the two bodies are equal to integral multiples of h, according to equations (3).

No difficulty arises from the fact that we have been dealing with a degenerate system, in which the numbers n' and k have no direct physical meaning, for we can justify our results by using relativistic mechanics; the system is not in reality degenerate.

2. Quantum Conditions for the Unexcited State of the Neutral Helium Atom

In our first calculation we neglect the motion of the nucleus, considering its mass as infinite. We use polar coordinates in accordance with the foregoing considerations, denoting them by r_1, φ_1, r_2, φ_2 for the two electrons, respectively. In formal analogy to equations (3) we assume the quantum conditions,

$$\begin{cases} \int p_{r_1} dr_1 + \int p_{r_2} dr_2 = n'h \\ \int p_{\varphi_1} d\varphi_1 + \int p_{\varphi_2} d\varphi_2 = kh \end{cases} \tag{11}$$

The natural, unexcited state of the atom will be characterized by the least possible values of n' and k. Neither n' nor k can be zero, for all phase integrals are positive quantities. If n' were zero, both radial integrals would vanish, and each electron would move upon a circular orbit. Because of the numerical equality of the angular momenta of the electrons, the circles would have the same radius and the electrons would collide. On the other hand, if k were zero, both azimuthal phase integrals would vanish, the electrons would move on straight

lines, and would collide with the nucleus. So the least possible quantum numbers are

$$n' = 1, \quad k = 1.$$

Since both electrons are equivalent in every way, we split up our two equations (11) into the four equations

$$\int p_{r_1}\, dr_1 = \int p_{r_2} dr_2 = \int p_{\varphi_1} d\varphi_1 = \int p_{\varphi_2}\, d\varphi_2 = \tfrac{1}{2}\, h. \tag{12}$$

These equations are exact, in so far as the motion of the nucleus may be neglected. There are two constants of integration for each coordinate, making a total of eight. The initial values of the time and of *one* of the azimuthal angles φ_1, φ_2, are arbitrary, so the number of constants is reduced to six. Whatever be the character of the orbits, —which are complex because of the mutual perturbation of the two electrons,—we can determine four of the constants of integration by means of eqs. (12). A seventh constant is given by the equation of angular momentum,

$$p_{\varphi_1} + p_{\varphi_2} = p = 0.$$

Only one more condition is required to define our model completely. This is supplied by the requirement that the period of the coordinate r_1 shall be the same as that of the coordinate r_2, in spite of their different initial values and the perturbations which exist. This period, τ_r, defines the interval of integration for the first two integrals of (12). The range of integration for the two azimuthal integrals is the time τ_φ in which φ_1 and φ_2 change by $\pm 2\pi$. Complete periodicity will occur only if $\tau_r = \tau_\varphi$.

If we wish to take account of the motion of the nucleus (to which we assign the polar coordinates R, Φ, referred to the center of gravity), we write instead of (11) the complete conditions

$$\begin{cases} \int p_{r_1} dr_1 + \int p_{r_2} dr_2 + \int p_R dR = n'h \\ \int p_{\varphi_1} d\varphi_1 + \int p_{\varphi_2} d\varphi_2 + \int p_\Phi d\Phi = kh \end{cases} \tag{13}$$

In the unexcited state ($n' = k = 1$) we get two more conditions by equating the first two integrals in each equation of (13), but these integrals no longer have the value $\frac{1}{2}h$. There are four equations describing the motion of the center of gravity, and one equation stating that the total angular momentum vanishes. Thus we have 9 conditions to be satisfied, while the number of integration constants is $3 \times 4 - 2 = 10$. Just as in the former case, we determine the remaining constant by the requirement that r_1 and r_2 have a common period.

The weak point in this treatment is our choice of polar coordinates. To justify it, we return to the case where the nucleus is supposed to be at rest, and take note of the fact that one of the azimuthal angles,

φ_1, φ_2, is a cyclic coordinate, and does not appear in the equation of energy. The other angular coordinate is then taken to be $\varphi_2-\varphi_1$. Thus, one azimuth plays a specialized rôle by virtue of the nature of the problem. This justification is of course very incomplete; we shall simply refer to the formal analogy with the problem of hydrogen as discussed in §1.

3. First Approximation, Elliptical Orbits

As a starting point for the solution of our dynamical problem, we neglect the mutual repulsion of the electrons, so that each of them describes a Keplerian ellipse. The major and minor axes, a and b, of such an ellipse depend only on the quantum numbers n and k assigned to the electrons ($n=k+n'=$principal quantum number), on a_1, the radius of the innermost circle of hydrogen, and on the atomic number Z.

$$a=n^2\,a_1/Z, \quad b=nk\,a_1/Z. \tag{14}$$

In the present case, $Z=2$, $n=1$, $k=\frac{1}{2}$. It is true, we do not assign quantum numbers, in the original sense of the word, to a single electron; the real quantum numbers n' and k in (11) belong not to a single electron, but to the whole system. But we see from eqs. (12) that it is convenient to assign partial quantum numbers to the electrons; thus,

$$n_1'=k_1=\tfrac{1}{2}; \quad n_2'=k_2=\tfrac{1}{2},$$

for the 1st and 2nd electrons, respectively, and these are the quantities we must use in (14) to obtain the size, and the eccentricity, ϵ, of the two elliptical orbits.

$$a=\tfrac{1}{2}a_1, \quad b=\tfrac{1}{4}a_1, \quad \frac{b}{a}=\sqrt{1-\epsilon^2}=\tfrac{1}{2}, \quad \epsilon=\frac{\sqrt{3}}{2}. \tag{14'}$$

In this first approximation we can say nothing about the relative orientation of the two orbits, or the simultaneous positions of the electrons upon the orbits. Nevertheless, in Fig. 1 we have drawn the

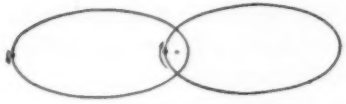


FIG. 1. *Model of helium atom.*

ellipses with their aphelia in opposite directions from the nucleus. One electron is taken to be at perihelion when the other is at aphelion, so that collisions are avoided and symmetry and periodicity are furthered as much as possible. The perturbations will affect not only

the shape of the orbits but also their instantaneous position with respect to each other. However, we may be certain that the average position of the two orbits will not be changed systematically, without limit, by the perturbations, because in the 2d half of any complete period of the motion the 2d electron is subjected to the same series of influences which act on the 1st electron during the 1st half of the period.

Millikan[6] has made an interesting study of ionization by α-particles. He found that helium atoms could be doubly-ionized in this way, though other atoms lost but a single electron. This seems understandable on the basis of the model here set forth. Millikan also showed that the ratio of doubly-ionized to singly-ionized helium atoms is about 1 to 6, which may be interpreted to mean that the time during which the electrons are in conjunction is $\frac{1}{6}$ of the time during which they are practically in opposition. In a general way this also seems to be in accord with the model shown in Fig. 1, because the electron which is at perihelion will move more swiftly than the electron which is at aphelion, in accordance with the theorem of areas, and so will soon arrive at opposition.

4. The K-Shell of the Heavier Elements

In the helium atom the perturbation of one electron, due to the repulsive force from the other, is very considerable; but if we deal with the K-shell of heavier atoms, which we assume to be similar to the configuration of helium in its most stable state, the perturbation continually becomes less important, because of the increasing nuclear charge. The relativity correction is proportional to the fourth power of the atomic number, and may be of greater magnitude than the electronic perturbation in the case of the heavier elements. So we may treat the two electrons in the K-shell as approximately independent.

But a difficulty arises; the relativity correction for this model of the K-shell is larger than it would be if the K-orbits were circles of the type 1_1. The first term in the general formula for the relativity correction to an orbit of the type n_k is,

$$\frac{a^2(Z-s)^4}{n^4}\left(\frac{n}{k}-\frac{3}{4}\right).$$

If $n=1$, $k=\frac{1}{2}$, the contribution of this expression to the frequency is

(15) $$\tfrac{5}{4}\ a^2\ (Z-s)^4$$

[6] Phys. Rev., *18*, p. 456; 1921.

for each of the two electrons. If $n=1$, $k=1$, we get

(16) $$\tfrac{1}{4}\,a^2\,(Z-s)^4$$

The contribution to the K-absorption limit would be

$$2\cdot\tfrac{5}{4}\,a^2\,(Z-s)^4-\tfrac{1}{4}\,a^2\,(Z-s)^4$$

because we have 2 K-electrons in the initial state on orbits of the type $n'=k=\frac{1}{2}$; while in the final state, after removing one electron, we have the remaining electron on a 1_1 orbit. Thus the contribution is

(17) $$\tfrac{9}{4}\,a^2(Z-s)^4.$$

This is 9 times higher than the value (16), which applies if both electrons in the normal K-shell are on 1_1 orbits. The expression (16) is in general agreement with observation, so that (17) proves to be too large. Notwithstanding, I believe that our general idea of the K-shell is correct, and would attribute this discrepancy to the rough character of our computations. Indeed, in the usual derivation of the relativity correction we assume that the orbit carries out its precession in the sense of the revolution. But the two electrons of helium revolve in opposite senses, and cannot precess freely; each prevents the other from so doing. From this we may expect, that our value (17) for the relativity correction is not applicable to our system of two electrons. Generally speaking, it seems to me impossible that there should exist in the interior of every atom a system possessing angular momentum and magnetic moment, such as Bohr's model of the K-shell.

All the foregoing considerations must remain incomplete and vague until we introduce quantitative information about the mutual perturbation of the two electrons. I decided to publish this paper only upon receipt of a letter from Mr. W. Heisenberg, who has solved in detail the dynamical problem of our model for unexcited helium, and has found that the ionization potential is about 24.5 volts, which is correct. I desire also to remark that a model of helium with half quantum numbers was mentioned by Bohr, but was considered by him to be inadmissible, in a conversation I had with him last summer in connection with the half-quantum magneto-optic model of Heisenberg.

UNIVERSITY OF MUNICH

Interpretation of Complicated Spectra (Manganese, Chromium, etc.) by the Method of Inner Quantum Numbers.—Let k be the azimuthal quantum number of a valence electron in one of its orbits. In every stationary state of the atom the *total* angular momentum is a multiple of $h/2\pi$, say $jh/2\pi$ where j is the inner quantum number. The changes of k and j during radiation are restricted by the selection principles, $\Delta k = \pm 1$, $\Delta j = \pm 1$ or 0; and for j the transition $0 \rightarrow 0$ is forbidden. Let $j = j_0$ for the unexcited state and let the valence electron be raised to a higher energy level, while receiving the angular momentum $j_1 h/2\pi$. If the final state is a stationary state of the atom, the vector $(j_0 + j_1)\ h/2\pi$ must be a multiple of $h/2\pi$. The vector $j_1 h/2\pi$ must be added only in a discrete set of directions, satisfying this condition, and the number of directions corresponding to a particular value of j_1, will be the number of spectral energy levels having the azimuthal quantum number $k_0 + j_1$. Examples:

Hg; $j_0 = 0$ for $1S$. Cr; $j_0 = 2$ for $1S$.

	j_1	j		j_1	j
P	1	1	$P_1 \ldots P_3$	1	3 2 1
D	2	2	$D_1 \ldots D_5$	2	4 3 2 1 0
B	3	3	$B_1 \ldots B_5$	3	5 4 3 2 1

Starting from the $1s$ state of Hg, $(j_0 = 1)$, we get three p-levels, three d-levels, etc. The general rule is seen from the table. *The degree of multiplicity of the energy levels remains constant after the appearance of a level for which $j = 0$.* These ideas are developed from the Mn, Mn^+ and Cr lines classified by Catalan. For example, the combination of the three P-terms with the five D-terms in Cr, yields a multiplet of 9 lines; the selection principles rule out the other 6. All observed multiplets of both elements are explained perfectly. For unexcited Cr, $j_0 = 2$; for Mn, we know only that $j_0 \geq 3$. Each unit of angular momentum corresponds theoretically to one Bohr magneton, so the magnetic moments of atoms may be predicted from the number of energy levels in their spectra. The value $j_0 = 2$ for Cr does not agree with the value $j_0 = 5$, obtained from experiments on the magnetic susceptibilities of chromium ions in solution, the explanation of which is not known at present.

Doublet spectra do not fit the above scheme. Here the Heisenberg model attributes $\frac{1}{2}$ unit of angular momentum to the valence electron and $\frac{1}{2}$ to the residue of the atom in the $1s$ state $(j_0 = 1)$. Adding $j_1 = 1$ we obtain for the mp_2 level, $\frac{3}{2} + \frac{1}{2} = 2$; and for mp_1, $\frac{3}{2} - \frac{1}{2} = 1$, and so on. In a similar manner the neon energy levels are attacked, but the agreement is not perfect, and we must wait for more observational material. [A. Sommerfeld, Ann. der Physik, *70*, p. 32, 1923.]

ARTHUR E. RUARK

SUMMARY OF INVESTIGATIONS IN THE INFRARED SPECTRUM OF LONG WAVE LENGTHS*

By W. Weniger

This report is a brief summary of the methods employed and the results obtained before 1922 in a study of the spectral region included in the wave-length interval, 50μ to 300μ.[1]

Apparatus

The apparatus used for investigations in the far infra red is, essentially, as for other regions of the spectrum, either a spectrometer or an interferometer. Prism spectrometers are out of the question because no substance is known that has a sufficient transmission.

The wire grating spectrometer has long been the favorite means of measuring wave lengths in this as in other regions of the spectrum, but until the work of Witt (1) its use was confined to wave lengths shorter than 32μ. Witt studied the region from 80 to 160μ with a grating made by wrapping silver wire, 0.17 mm in diameter, about two screws having a pitch of 0.3 mm, and cutting away the wire in one plane. He could use this apparatus at 3.5μ, but usually filtered the radiation through a quartz plate, thus absorbing practically all of the radiation from 4.5 to 50μ.

The grating of course is wasteful of energy and the spectrometer method necessitates the use of comparatively narrow slits. In an interferometer both of these disadvantages are avoided, and though there must necessarily be transmission through some substance, only thin layers need be used. The purity of the spectrum, however, is poor (2). In this field the interferometer has been used in connection with two schemes: (a) the method of residual rays, i.e., the method of isolating particular wavelengths by means of multiple reflection from selectively reflecting mirrors; (b) the method of extrafocal isolation, i.e., the method of separating the long from the short wave length regions by a quartz lens and suitable screens, such a separation being possible

* [Prepared at the Request of the Committee on Spectroradiometry. W. W. Coblentz, Chairman.]

[1] Our knowledge of the infrared spectrum, in the long wave lengths, is mainly owing to the labor and inspiration of one investigator, the late Prof. Heinrich Rubens. An appreciative summary of his life work, by some of his students, is given in Die Naturwissenschaften: Dec. 1, 1922. W. W. C.

because quartz has very different indices of refraction for wave lengths lying on either side of its absorption region (3μ to 50μ).

The arrangement of apparatus for obtaining residual rays as used by Rubens and Hollnagel (3), is illustrated schematically in Fig. 1,

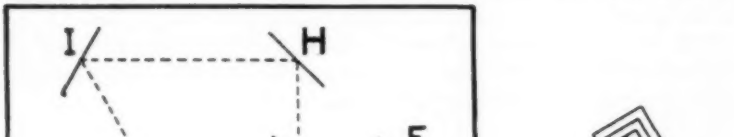


FIG. 1. *Arrangement of apparatus for obtaining residual rays by multiple selective reflection and for analyzing them by means of an interferometer C.*

in which the concave mirror *B* focuses the radiation from the source *A*, upon an interferometer of thin quartz plates at *C*. The shutter *D* of rock salt is opaque to the radiation of wave length greater than 30μ, which is under investigation. The selectively reflecting mirrors (of rock salt, sylvite, etc.) for obtaining residual rays are placed at *E*, *F*, *G*, and *H*. These rays are focused upon the radiometer *J*, by means of the concave mirror *I*.

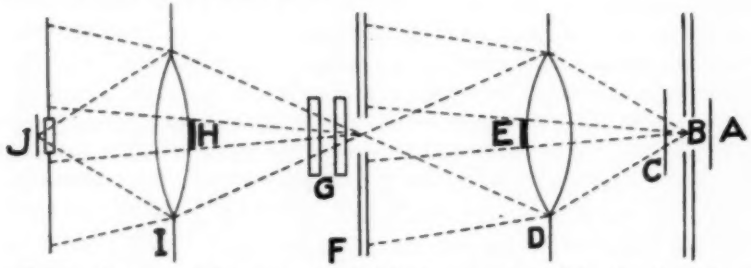


FIG. 2. *Arrangement of apparatus for obtaining long wave length infrared radiation by extrafocal isolation.*

The method of extra focal isolation of long wave-length infrared radiation, illustrated in Fig. 2, was first used by Rubens and Wood (4) and was slightly improved by Rubens and Baeyer (5). The principle on which it is based was published by Rubens and Aschkinass (6).

In Fig. 2, A is the source of radiation; B and F are metal screens of sheet iron with openings 15 mm in diameter, and C is a plate glass shutter. The quartz lenses D and I are 10 cm in diameter, 5 mm thick at the edge and 12 mm thick at the center; the focal length for visible rays being 25 cm. At E is a disk of black paper about 25 mm in diameter. The interferometer is at G and the radiometer is placed at J.

In layers of several millimeters, quartz is transparent to the near infrared, and for this region its refractive index lies between 1.55 and 1.43. These rays form a divergent cone after passing the lens and are intercepted by the screen F. The central part of this cone is prevented from passing through the opening in F by the black paper E. For the far infrared, quartz has a refractive index of about 2.14. These rays are brought to a focus at the air film of the interferometer G. Some of the short wavelengths do, however, reach the interferometer on account of scattering, mainly at the surface of the lens D. As the energy in the scattered rays is large compared to the energy in the long wave lengths, it must be reduced before it reaches the radiometer. This is accomplished by introducing the lens I and repeating the process of separation.

The interferometers used consist merely of two thin pieces of quartz perhaps 5.5 cm in diameter and 0.6 mm in thickness, one plate being mounted rigidly and the other mounted on a screw carriage with adjustments for making it parallel to the fixed plate.

For the intensity of a homogeneous radiation after its passage through a plane parallel air space, Airy's formula gives

$$I = I_0 \frac{(100-R)^2}{(100-R)^2 + 400\,R \sin^2 (2\pi d/\lambda)}$$

where I is the intensity of radiation, I_0 the maximum intensity (amplitude), R the reflection factor at normal incidence, λ the incident wave length, and d the thickness of the air film. For quartz, if $I_{max} = I_0$, $I_{min} = 0.574\ I_0$. If the radiation is non-homogeneous, the difference between I_{max} and I_{min} will be less than indicated. According to this formula, a change in the thickness of the air film will cause alternate maxima and minima to be recorded by the radiometer, J. With radiation of two wavelengths, the maxima will not decrease regularly as the thickness of the air film increases, but will vary periodically. If the curve never flattens out to a straight line, the two components are not of the same amplitude, and their relative intensities can be determined (3), (7), (8), (9).

The radiometer which has been used by most investigators is the Boy's radiometer micrometer (1), (10), (11); but a thermopile and iron-clad Thomson galvanometer could be used.

The source of radiation is all important, and quite naturally a great many different types have been examined for possible long wave emission. Sparks between metal terminals have not proved fruitful. The black body and the Nernst glower have been used to some extent.

The Welsbach mantle without a glass chimney is very convenient up to 100 or 110μ, and is commonly employed. An extremely long wave length region (300μ) is emitted by the mercury vapor in a quartz enclosed arc lamp (12), (5).

The radiation from the mercury arc was filtered through two mm of amorphous quartz or through 0.38 mm of black cardboard to absorb the radiation due to the hot quartz of the lamp, the latter being very similar to the radiation from the Welsbach mantle. The radiation is complex, being composed of two rather wide beams, of wave lengths 218μ and 343μ, giving a mean wave length for the transmitted beam of 313μ or nearly $\frac{1}{3}$mm, as determined by the method of extra focal radiation.[2]

PROPERTIES OF MATERIALS IN THE REGION OF LONG WAVE LENGTHS

The problems that have been attacked include the determination of wave lengths, reflection factors, absorption factors, extinction coefficients, and transmission factors. Attempts have also been made to incorporate the results in atomic models (17). A brief survey of the results obtained follows.

WAVE LENGTH DETERMINATIONS. The reflecting surfaces were prepared in a number of different ways: natural crystals cleaved and polished; small pieces of natural crystals mounted on a plate and polished; substance fused and cast; casts turned in lathe; crystals pressed together. The wave length of the residual rays was obtained by successive reflection, as illustrated in Fig. 1.

A very homogeneous radiation of $\lambda = 27.3\mu$ is obtained by reflecting the radiation from a Welsbach mantle twice from calcspar, twice from fluorspar, and filtering through 3 mm of potassium bromide (18).

[2] As this paper goes to press new investigations on short electric waves from the quartz mercury arc are being published by Nichols and Tear, (Amer. Phys. Soc. Boston Meeting, Dec. 27 to 29, 1922).

TABLE 1. *Wave lengths of Residual Rays from Various Substances*

Substance	Wave Lengths, μ		References
	Actually Observed	Mean	
Rock salt, $NaCl$	47, 54	52.0	(3) (6) (13) (7) (14)
Sylvite, KCl	62, 70	63.4	(3) (6) (7) (14)
Potassium bromide, KBr	74, 86	82.6	(3) (7)
Potassium iodide, KI		94.1	(3) (7)
Silver chloride, $AgCl$	74, 90	81.5	(7) (16)
Silver bromide, $AgBr$		112.7	(7) (16)
Calomel, Hg_2Cl_2	91, 118	98.8	(16)
Lead chloride, $PbCl_2$	74, 92, 114	91.0	(7) (16)
Calcspar, $CaCO_3$	93, 116	98.7	(15)
Sal ammoniac, NH_4Cl	46, 54	51.5	(13) (17)
Ammonium bromide, NH_4Br	55, 62	59.3	(17)
Thallium chloride, $TlCl$		91.6	(17)
Thallium bromide, $TlBr$		117.0	(17)
Thallium iodide, TlI		151.8	(17)
Corrosive sublimate, $HgCl_2$		95.	(17)
Silver cyanide, $AgCN$		93.	(17)

TABLE 2. *Transmission of Quartz* (17). *The Values Given are for Unpolarized Light and do Not Take into Account Reflection from Surface. Temperature of Specimens 10-12°C*

Residual Rays From	Wave Length, μ	Plate Cut $\perp$ Axis $d = 1.93$ mm	Plate Cut $\perp$ Axis $d = 3.99$ mm	Plate Cut $\perp$ Axis $d = 7.26$ mm	Plate Cut $\parallel$ Axis $d = 7.39$ mm
NH_4Cl	51.5	25.1%	9.83%	2.90%	4.01%
$NaCl$	52.0	28.2	10.9	2.94	4.23
NH_4Br	59.3	36.8	17.7	6.56	8.95
KCl	63.4	41.1	21.9	8.48	11.5
$AgCl$	81.5	49.1	32.8	19.2	24.1
KBr	82.6	48.5	32.6	19.1	24.4
$PbCl_2$	91.0	57.8	44.1	31.3	33.9
$TlCl$	91.6	59.2	46.4	33.9	35.6
$AgCN$	(93)	61.4	48.8	35.8	36.3
KI	96.7	61.4	50.3	36.1	37.8
$HgCl_2$	(95)	61.8	49.8	37.8	38.8
$CaCO_3$	95.7	....	....	40.3	....
Hg_2Cl_2	98.8	63.2	52.7	40.5	41.6
$AgBr$	112.7	68.7	61.5	50.8	51.7
$TlBr$	117.0	69.2	63.7	53.9	55.0
TlI	151.8	....	65.0	58.0	....

TRANSMISSION FACTORS. To take transmission curves, the interferometer is removed and an amount of quartz equal to the combined thickness of the interferometer plates is inserted so as not to change the energy distribution. The transmission of quartz is given in Table 2; and that of various other substances for long wave length radiation obtained by extra focal isolation is given in Table 3.

TABLE 3. *Transmission of Long Wave Length Radiation Through Various Substances (Not Corrected for Reflection)*

Substance	Thickness mm	Source of Radiation			
		Welsbach mantle 107 μ	*Hg* Arc	*Hg* Arc Filtered through 2.0 mm of Amorphous Quartz	*Hg* Arc Filtered through Black Cardboard 0.38 mm thick 310μ
		%	%	%	%
Quartz cut $\perp$ axis	41.7	12.1	25.4	51.8	58.9
Amorphous quartz	2.0	12.5	24.2		60.0
Fluorite	0.59	5.3	19.4	39.5	42.2
Rocksalt, cleaved	1.29	0.5	5.7	16.5	22.5
Sylvite	2.10	0.0	3.6	11.7	16.7
Diamond	1.26	45.3	64.5		
Selenium	0.52	6.8	12.9	24.2	
Mica	0.055	16.6	38.8	51.5	55.0
Glass	0.18	2.1	9.0	21.1	25.9
Paraffin	3.03	57.0	72.3	82.5	85.5
Hard rubber	0.40	39.0	51.5	58.8	65.3
Amber	1.28	11.2	16.4	32.2	34.8
Celluloid	0.26	16.2	27.6	38.7	43.5
Water	0.019		55.5	60.3	62.7
Water	0.038	20.0	33.0	38.4	39.8

Quartz is dichroic between 53μ and 110μ in the sense that radiation is more strongly absorbed when the electric vector is perpendicular than when it is parallel to the axis (7).

Water vapor has absorption bands at 50, 66, and 79μ and probably also at 58 and 103μ. It has regions of high transmission at 47, 54, 62, 75, 91 and 115μ (7). There seem to be additional regions of high transmission at 125 and 138μ.

From the standpoint of laboratory practice the transmission of black paper and lamp black, given in Table 4, is of interest, (4), (7), (12).

TABLE 4. *Transmission of Black Paper and Lampblack for Various Wave Lengths*

Source of Radiation	Wave length in μ	Per cent Transmission			
		Black Tissue Paper	Black Paper from Photographic Plates	Black Cardboard 0.4 mm thick	Lampblack
Radiation Isolated with mirror spectrometer and *KCl* or fluorite prism	2	0	0	0	0.5
	4	0.9	0	0	8.6
	6	1.7	0	0	16.0
	12	8.2	1.4	0	37.6
Reststrahlen fluorite	26	24.2	3.2	0	76.7
Reststrahlen *NaCl*	52	46.0	15.1	0	91.3
Welsbach Mantle andQuartz lens	108	61.5	33.5	1.6	91.5
Mercury arc total			52.4	11.7	
Mercury arc filtered through 2 mm of amorphous quartz			78.5	29.8	
Mercury arc filtered through 0.4 mm of black cardboard	310μ		79.0	36.7	

REFLECTION FACTORS. To measure reflection factors, Table 5, the interferometer is usually replaced by two 45° silver mirrors and a hori-

TABLE 5. *Reflection Factor for Various Substances*

Substance	Reflection factor, R		Dielectric Constant, K	R_∞
	KBr residual rays 82μ	Welsbach Radiation Isolated by Lens: 108μ		
	%	%		%
Calcspar		47.1	8.5	24.0
Marble		43.8	6.14	18.1
Rock Salt	25.8	20.3	6.29	18.5
Sylvite	36.0	19.3	4.94	14.3
Potassium Bromide	82.6	31.1		
Potassium Iodide	29.6	35.5		
Fluorite	19.7	20.2	6.8	19.7
Glass		19.2	6.66	19.5
Water	9.6	11.6	81.0	64.0
Alcohol		1.6	25.0	44.5
Castor oil		4.3	4.78	13.9

TABLE 6. *Reflection and Transmission Factors for Various Substances*

Crystal and Source	Plane of Vibration of Electric Vector	Reflection factor, R, for residual rays from:								R, Extrafocal Radiation from Quartz Lens:			Transmission Factor		R_∞ Computed
		CaF_2	CaF_2, $CaCO_3$ (Calcspar)	CaF_2	$CaCO_3$ (Aragonit)	$NaCl$	KCl	KBr	KI	Welsbach Burner	Mercury Arc: Total	Mercury Arc: Filtered	$D_{310\mu}$ Computed	D_∞ Observed	
Wave Length		22μ	27μ	33μ	39μ	52μ	63μ	83μ	94μ	110μ		310μ			
Lead nitrate		6.9	6.2	5.5	3.1	5.0	29.4	63.7	55.8	50.8	41.5	36.9	16.8	16.0	36.0
Analcite (Seisser Alp)		23.9	18.8	16.8	14.5	12.9	12.7	11.8	12.4	16.1	18.5	19.7	6.7		
Caesium Alum		6.5	4.2	4.4		18.9	14.3	14.6	13.0	14.6	14.6	14.6	5.0		
Rubidium Alum		6.4	4.8	5.1		13.9	16.0	14.1	13.1	15.0	15.0	15.0	5.1		
Rubidium Chrome alum		6.6	5.1	7.0		14.8	13.6	15.6	14.1	14.5	14.5	14.5	5.0		
Ammonium Alum		6.5	4.4	5.7	9.2	18.9	18.0	16.0	15.1	17.7	17.8	17.8	6.0		
Opal (Kaschau)		35.6	15.7	12.9		10.7	10.6	10.6	11.1	11.6	11.3	11.0	4.0		
Opal (Mexico)		30.6	13.9	11.7		9.7	10.0	10.2	10.4	11.0	11.0	11.0	4.0		
Chalcedony (Iceland)		33.1	24.2	18.0		14.2	13.6	12.7	12.8	13.0	12.2	11.8	4.2		
Quartz glass		34.0		13.0		11.6	11.1	10.9		10.5	10.3	10.2	3.8	3.75	10.2
Quartz, ordinary ray		59.3		16.8		14.3	13.9	13.3		13.0	12.8	12.7	4.4	4.44	12.7
Quartz, extraordinary ray		24.3		20.2		15.5	14.8	14.4		13.9	13.7	13.6	4.7	4.65	13.4
Wurzite (Pribram)		7.5	33.4	64.8	46.2	28.3	26.5	24.0	23.8	23.3	23.4	23.4	8.2		
Zincblend (Diez, Nassau)		7.2	35.4	73.7	51.9	30.3	27.7	25.5	25.0	24.4	23.8	23.5	8.3	7.85	22.5
Zirkon I (Frederiksvarn)	‖	39.6	52.5	43.7	32.5	25.5	24.3	23.5	24.1	24.3	24.4	24.4	8.7	12.6	31.4
	⊥	25.2	37.4	51.5	42.1	26.0	23.1	21.9	27.8	24.5	23.9	23.6	8.4	11.8	31.8
Zirkon II (Frederiksvarn)	‖	37.7	62.1	42.8	35.3	27.8	26.7	25.0	26.9	27.3	27.6	27.8	10.4	12.6	31.4
	⊥	23.7	43.6	56.5	44.3	28.4	26.1	24.8	28.7	28.4	28.3	28.2	10.7	12.8	31.8
Rutile (Graves Mountain)	‖	92.6	94.7	95.3	93.9	90.3	85.4	78.3	76.7	75.5	74.0	73.3	167	173	73.6
	⊥	41.2	77.4	92.4	95.2	88.1	79.9	68.1	66.4	65.7	64.8	64.4	83	89	65.1
Strontianite (Drensteinfurt)	‖	1.5	1.4	18.9	48.0	62.0	42.5	25.5	22.1	20.3	19.5	19.1	6.5		
	⊥	2.4	2.8	24.9	50.5	39.0	28.2	23.5	22.1	20.5	19.7	19.3	6.6		
Cryolite (Ivigtut)	‖	4.6	4.6	8.5	21.8	26.9	41.1	18.4	18.2	25.6	24.4	23.8	8.5		
	⊥	4.4	4.8	10.0	18.2	31.2	30.3	19.0	17.7	19.8	17.8	16.8	5.7		
Tourmaline (Uruienga)	‖	32.6		18.8		17.8	17.4	16.8		18.1	17.9	17.8	6.0	5.6	16.6
	⊥	32.0		24.4		22.4	20.8	21.2		22.4	21.2	20.6	7.1	6.8	19.8
Tourmaline (Modum)	‖	33.7		17.2		16.2	15.8	17.7		18.1	18.1	18.1	6.2		
	⊥	33.9		24.4		22.6	21.3	22.0		22.2	21.5	21.1	7.3		

TABLE 6. (continued)

TABLE 6. (continued)

Crystal and Source	Plane of Vibration of Electric Vector	Reflection factor, R, for residual rays from:								R, Extrafocal Radiation from Quarts Lens:			Transmission Factor		R_∞ Computed
		CaF_2	CaF_2 $CaCO_3$ (Calcspar)	CaF_2	$CaCO_3$ (Aragonit)	$NaCl$	KCl	KBr	KI	Welsbach Burner	Mercury Arc: Total	Mercury Arc: Filtered	$D_{114\mu\mu}$ Computed	D_∞ Observed	
Tourmaline (Haddam)	∥	32.4		18.0		15.8	15.6	15.5		17.7	18.1	18.3	6.2		
	⊥	31.6		23.7		20.9	19.4	18.8		22.0	21.0	20.5	7.1		
Red Tourmaline (Schaitansk)	∥	29.2		14.4	19.9	17.3	15.2	16.8	19.0	18.3	18.0	17.8	6.0		
	⊥	32.3		22.1	24.7	23.1	17.1	22.1	21.5	21.3	20.4	20.0	6.9		
Beryl (Nertschinsk)	∥	59.3	33.5	20.5	19.0	18.6	18.6	16.6	16.7	18.3	18.0	17.8	6.0	5.5	16.2
	⊥	35.7	20.8	18.3	17.3	17.2	16.4	16.4	17.1	17.7	17.6	17.5	5.9	6.1	18.0
Beryl (S. W. Africa)	∥	58.3	33.3	21.0	18.2	18.6	18.2	16.5	18.2	18.8	17.8	17.3	5.8		
	⊥	35.6	20.9	18.3	17.6	16.9	16.5	16.5	17.8	18.5	18.0	17.8	6.0		
Tin oxide (Schlaggenwald)	∥	54.0	35.3	33.8	31.6	30.9	30.8	29.0	32.3	41.5	42.9	43.6	24.0		
	⊥	17.8	47.6	72.7	60.5	39.8	36.8	35.0	38.9	41.5	42.6	43.1	23.4		
Vesuvian (Egg)	∥	38.8	24.8	24.6	22.6	22.4	25.2	30.3	28.4	27.0	26.2	25.8	9.4	8.9	24.8
	⊥	29.6	28.7	28.8	26.6	29.3	27.1	24.1	25.1	24.7	24.3	24.1	8.6	8.4	23.7
Sodium salt peter	∥	1.7	0.6	6.6	21.0	22.2	11.1	4.1	2.9	36.7	37.6	38.0	17.8	5.18	15.2
	⊥	3.5	2.4	8.7	26.5	29.9	18.1	11.7	8.5	20.9	19.7	19.1	6.5	5.18	15.2
Iron carbonate (Ivgtut)	∥	5.7	30.8	20.2	30.2	69.7	41.5	25.7	22.5	21.1	18.8	17.7	6.0	6.9	20.2
	⊥	5.5	42.4	46.5	31.0	39.1	31.7	25.9	24.3	23.5	22.7	22.3	7.8	7.9	22.4
Zinc carbonate (Laurion)	∥	2.2	15.4	20.4	29.0	79.0	75.9	41.5	31.9	29.6	27.0	25.7	9.4		
	⊥	3.7	21.7	45.5	32.6	59.5	50.4	33.8	31.7	29.5	26.8	25.4	9.3		
Sodium tripotassium sulphate	∥	3.6	1.9	7.0	19.9	23.5	35.0	23.8	17.5	19.4	17.9	17.2	5.8		
	⊥	3.3	1.7	6.4	15.9	34.2	39.0	17.3	14.2	21.0	20.9	20.9	7.2		
Potassium lithium sulphate	∥	15.0	29.4	14.1	8.6	5.1	12.4	26.5	22.2	19.5	17.6	16.7	5.7		
	⊥	21.0	16.3	10.6	6.1	4.4	15.0	24.4	19.0	18.3	16.6	15.9	5.4		
Topaz (Alabasca)	a	57.2	29.2	24.6	22.1	20.2	20.5	19.9	20.4	20.9	21.2	21.4	7.4	6.7	19.6
	b	53.2	33.9	27.6	25.9	22.6	21.1	20.5	21.0	21.5	21.3	21.2	7.3	6.7	19.6
	c	50.3	26.8	27.5	25.0	22.1	20.7	19.4	20.3	20.9	20.7	20.6	6.7	6.3	18.5
Witherite (Cumberland)	∥	2.1	1.1	12.9	40.4	86.0	78.9	37.7	26.1	24.4	22.2	21.1	7.3	6.4	18.8
	⊥	4.5	2.3	15.0	52.1	66.5	38.1	28.4	25.2	24.6	22.1	21.6	7.5	7.5	21.7
Malachite (Nischne Tagilsk)		8.5	15.5	32.4	23.6	20.0	23.5	24.2	24.0	22.2	21.3	20.8	7.2		
Adularia (001) (St. Gotthard)	∥	16.4	15.0	12.1	8.6	7.6	10.0	19.2	16.7	15.7	14.8	14.4	4.9		
	⊥	35.7	20.4	17.2	14.4	10.4	8.6	13.5	29.0	21.0	18.4	17.1	5.8	5.5	16.1
Spodumene (100) (Pala, California)	∥	53.7	23.7	26.0	42.5	38.1	33.2	26.6	26.5	28.6	28.2	28.0	10.5		
	⊥	37.3	21.2	45.0	45.0	36.0	30.0	22.4	25.4	25.2	25.1	25.1	9.1		

zontal mirror. The first 45° mirror sends the radiation downward upon the horizontal mirror at an angle of incidence of 15° or less, an image of the source being formed on the surface of the horizontal mirror. The other 45° mirror takes the radiation that proceeds from the horizontal mirror and sends it on to the selectively reflecting mirrors, used in producing residual rays, or to the second quartz lens, as the case may be. The horizontal mirror is alternately one of silver and one of the substance under investigation. Means must be provided for levelling and for bringing all mirrors to the same height.

R_∞ is calculated from $R_\infty = 100\left(\frac{1+\sqrt{K}}{1-\sqrt{K}}\right)^2$. The computed values for rock salt, fluorite and glass agree fairly well with the observed. The indices of refraction of water, alcohol and castor oil are of the same order of magnitude as in the visible.

Reflection factors and transmission factors have been obtained from a number of crystals, using plane polarized radiation (18) (23). The radiation is polarized by reflection from a selenium mirror (20). Some of these measurements are tabulated in Table 6. Where the direction of the plane of vibration is not specified, unpolarized light was used.

An interesting piece of work in the transmission and reflection of polarized radiation was done by Rubens (19) (20). In monoclinic crystals neither the optic nor the dielectric axes in a plane normal to the axis of symmetry can be determined from the geometric properties of the crystal. Each must be determined experimentally, as no relation is known between the position of the dielectric axes and the position of the optic axes in the visible. Electromagnetic theory, however, indicates that for long wave lengths, the two sets of axes should coincide. This was verified.

The substances experimented upon were adularia (K, Na) $Al\ Si_3\ O_3$ and gypsum. The dielectric axes were determined by means of Lichtenberg's figures (21) and by suspending the crystal between vertical condenser plates (22). The optic axes were determined by reflection, using the customary apparatus, but with polarized radiation from a mercury arc. The mirror was rotated in its own plane through an angle of 22.5° between observations.

The foregoing brief summary indicates the apparatus and methods required for isolating and measuring long wave length thermal radiation, as well as some of the results obtained by investigation of this interesting spectral region.

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Series in the Arc Spectrum of Lead.—Thorsen, working in Bohr's laboratory, has found the sharp and diffuse series in the arc spectrum of lead an especially interesting achievement inasmuch as the spectrum of no other element in this group has been correlated in series. The spectrum may be classed as widely spaced triplets quite different however from the triplet structure of the alkaline earths. There are three mp levels, mp_1, mp_2, and mp_3 and, speaking approximately, three md levels, md_1, md_2 and md_3. However the md_2 level is a very close doublet designated as md_2' and md_2''. The separation for $m=3$ is only 7 wave numbers and for higher values of m the separation was too small to be observed with the quartz spectrograph employed. Curious combinations occur producing satellites and composed triplets of an entirely new type. Thus $3d_1$ combines with $2p_1$ and $2p_2$ but not with $2p_3$; $3d_2'$ combines with $2p_1$ and $2p_3$ but not with $2p_2$; $3d_2''$ combines with $2p_2$ and $2p_3$ but not with $2p_1$; $3d_3$ combines with all the $2p$ terms. Still more curious, the above being true, is that $4d_1$ combines with all of the $2p$ terms. The following table gives the values of the variable terms. [V. Thorsen, (Copenhagen) Die Naturwissenshaften, Feb. 2, pp. 78–9; 1923.]

m	mp_1	mp_2	mp_3	md_1	md_2	md_3	ms
2	38362	49173	52005				24530
3				13485	13746 13753	14371	11126
4				7313	7501	7711	6301
5				4653	4727	4808	4091
6				3210	3246	3275	2871
7				2344	2361	2390	2123
8				1798		1812	

PAUL D. FOOTE

A Correction.—In L. W. McKeehan's paper: "Extraordinary Diffraction of X-rays" (This Journal, *6*, p. 993; 1922) Equation (a') should read as follows:

$$\lambda_1 = \frac{1}{\Sigma(e_1^2)}\left[a\Sigma(l_1e_1) - \lambda_2\Sigma(e_1e_2) \pm \sqrt{[a\Sigma(l_1e_1) - \lambda_2\Sigma(e_1e_2)]^2 - \lambda_2\Sigma(e_1^2)[\lambda_2\Sigma(e_2^2) - 2a\Sigma(l_1e_2)]}\right] \quad \ldots \quad (a')$$

INSTRUMENT SECTION

COUPLED OSCILLATIONS OF A HELICAL SPRING

By A. Sommerfeld

The Central Scientific Company of Chicago intends to reproduce a simple mechanical contrivance, which I proposed in 1905 in a paper published in "Festchrift Adolph Wüllner Gewidmet zum Siebzigsten Geburtstage," Leipzig, Teubner; 1905. This paper contains a full theoretical account of the problem. In the following lines I mention only a few points in order to illustrate the idea of the little instrument, shown in Fig. 1, and its use in determination of the elastic constants of the wire.

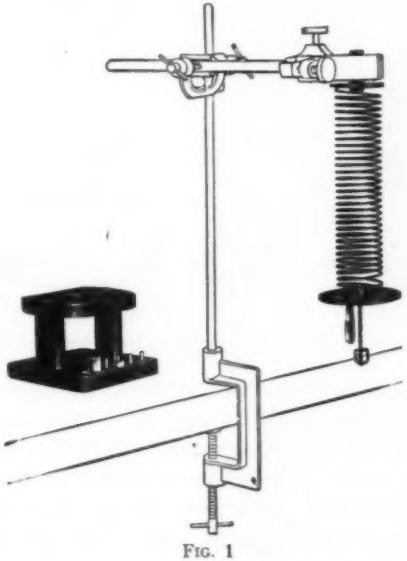

Fig. 1

A previous description of the dynamical phenomena shown by a helical spring is given by Wilberforce, Phil. Mag., *38*, 386, 1894. (See also Crew, "General Physics.") Comparing the following treatment with this previous account of the spring, the reader will see in particular that the application to the determination of elastic constants is here carried somewhat further.

1. TORSIONAL AND BENDING MOMENTS OF THE SPIRAL SPRING

If the free end of the spring is pulled downwards by a force Q acting along the central line of the spring (see Fig. 2), r being the radius of the spring, viz., the distance of the central line from the wire, then we have for any cross section of the spring a moment Qr, the axis of which is the center line of the wire, provided we neglect the small angle α of inclination of the wire with respect to the horizontal plane. This

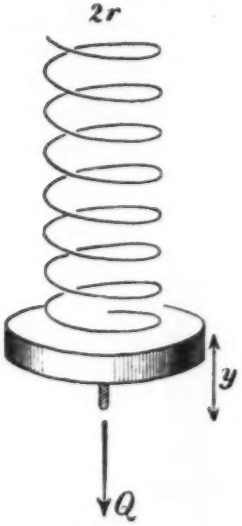

FIG. 2

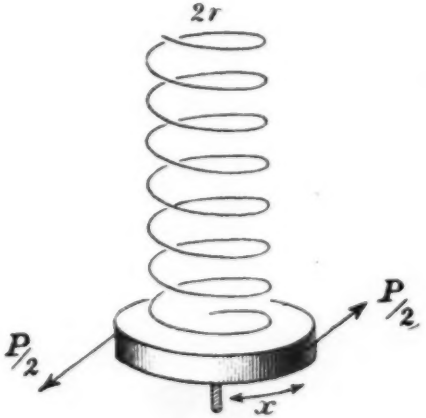

FIG. 3

is a torsional moment; and consequently the elastic constant of the wire, coming into play in the corresponding deformations, is the torsional modulus or modulus of rigidity n; and the moment of inertia of the cross section, which occurs in the expression of deformation, is the polar moment of inertia I_p of the cross section. If l is the length of the wire and y the displacement of the end of the spring parallel to the force Q, we have

$$y=\frac{Q\,r^2 l}{n\,I_p},\quad Q=\frac{n I_p}{r^2\,l}\,y. \tag{1}$$

On the other hand, if we apply at the end of the spring a pair of horizontal forces $P/2$ with the moment Pr (see Fig. 3), the axis of this moment is parallel to the central line and therefore parallel to any

cross section of the wire, provided we neglect the inclination angle α between the axis of the wire and the horizontal plane. Such a moment is a bending moment. So the elastic constant is now Young's modulus of elasticity e, and the moment of inertia of the cross section is the moment of inertia I of the cross section with respect to its diameter. Stressed by the moment Pr the wire yields in an horizontal direction. Its displacement x is given by the formula

$$x=\frac{Pr^2l}{e\,I}, \qquad P=\frac{eI}{r^2l}\,x. \tag{2}$$

As is shown in the theory of elasticity,

$$n=\tfrac{1}{2}\frac{e}{1+\mu}$$

where μ is Poisson's ratio. For a circular cross section, we have

$$I_p=2I.$$

Thus, we obtain

$$nI_p=\frac{eI}{1+\mu}$$

Therefore, we may write instead of (1).

$$y=\frac{Q\,r^2l}{eI}\,(1+\mu), \qquad Q=\frac{eI}{r^2l\,(1+\mu)}\,y \tag{3}$$

2. TORSIONAL AND BENDING OSCILLATIONS

In what follows we do not deal with statical problems or with externally applied forces, but with oscillations and the corresponding reactions of inertia. Therefore we have to replace Q and P in the equations (2) and (3) by

$$Q=-M\frac{d^2y}{dt^2}, \qquad Pr=-\Theta\frac{d^2\phi}{dt^2} \tag{4}$$

Here M means the effective mass taking part in the y-motion, viz. the whole mass of the disk attached at the end of the spring and 1/3 of the mass of the wire. Θ means the moment of inertia of the disk around the central line together with $r^2/3$ times the mass of the wire. We put

$$\Theta=M'r^2$$

and call M' the reduced mass, so that M' at the distance r from the central line gives the same moment of inertia as the real mass distribution at its different distances. ϕ is the angle of rotation; hence x equals $r\phi$. Therefore we have from (4)

$$Q = -M\frac{d^2y}{dt^2}, \qquad P = -M'\frac{d^2x}{dt^2} \tag{5}$$

and from (2) and (3)

$$\begin{cases} M'\dfrac{d^2x}{dt^2} + \dfrac{eI}{r^2l}x = 0 \\ M\dfrac{d^2y}{dt^2} + \dfrac{eI}{r^2l\,(1+\mu)}y = 0. \end{cases} \tag{6}$$

Let ω_x and ω_y be the proper frequencies of the two oscillations in 2π units of time. From (6) we deduce

$$\omega_x^2 = \frac{eI}{M'r^2l}, \qquad \omega_y^2 = \frac{eI}{Mr^2l\,(1+\mu)},$$
$$\frac{\omega_x^2}{\omega_y^2} = \frac{M}{M'}\,(1+\mu). \tag{7}$$

If we produce both oscillations x and y together, we observe a resultant motion known as Lissajous' curve. From (7) we see that the character of this curve of the component oscillations, which depends on the ratio of the periods, is mainly given by Poisson's ratio together with the ratio of masses M and M'.

3. Effects of Resonance

We may expect to find especially interesting phenomena if the two oscillations have the same or approximately the same free periods. Then we have the case of exact or approximate resonance. The two oscillations are tuned together or are only a little mistuned. The classical example of two oscillations in resonance is that of the so called sympathetic pendulums, two pendulums of equal length attached to the same flexible support or joined by an elastic spring. It is well known in such cases that if we excite one degree of freedom (the one pendulum: one of our x or y oscillations), then the other degree of freedom is excited by the coupling effects. The energy communicated to the first degree of freedom goes over to the second. In the case of perfect resonance, the whole energy is transformed, so that the oscillations of

the first degree of freedom come to zero amplitude, while those of the second degree go to a maximum. Then the opposite takes place: the second degree of freedom is the contributing part, the first one the accepting part, and the energy is transformed from the second to the first while there is no energy left in the second degree of freedom. This alternating play continues for a long while, if there is sufficiently small damping. If we have only approximate resonance and we excite at the beginning only the y oscillations, then the x oscillations increase at the expense of the y oscillations. The x oscillations reach a maximum, but meanwhile the y oscillations fall to a minimum of energy, not to zero amplitude as in the case of perfect resonance. If the energy is retransformed, the y oscillations reach their initial energy and consequently the x oscillations go to zero. This alternating play continues until all the energy has been dissipated through damping.

We observe these effects very nicely in our spring if we tune the two oscillations together. In order to do this, we place small weights upon the central axis of the spring. There are provided pieces of .5, 1, 2 and 5 grams. As they are attached on the central axis, they do not influence in an appreciable way the mass M' (the moment of inertia of the system) while they increase the mass M and thereby lengthen the period of the y vibration. This period is originally the shorter one; we are able to bring it nearer to the period of x-vibration and make the two periods coincide.

We have an especially exact estimate of the degree of resonance if we count the number of oscillations between two instants when one of the two oscillations is zero. This number N is a maximum if the resonance is perfect, and decreases on both sides, either making the x or the y oscillation period greater. We should expect, corresponding to the general notion of beats, that the relation

$$N = \frac{t}{t_x - t_y} \tag{8}$$

would hold, where t_x and t_y are the free periods of x and y oscillations and t the arithmetical mean of both. But equation (8) is correct only for cases which are sufficiently far from resonance. Indeed, in the case of resonance, all depends on the coupling of the two degrees of freedom. So it is necessary to correct the original equations (6) and to introduce therein factors corresponding to the condition that one oscillation is gaining on the other.

4. EFFECTS OF COUPLING

If we have as in §1 a force Q in the central line, working downwards, the spring will not only be lengthened by an amount y but it will also yield backwards by a certain amount x. On the other hand, if we work with a peripheral force P, the spring will not only follow in the direction of the periphery but it will yield somewhat at the same time in the axial direction. The amount of this reaction in both cases depends obviously on the amount of the angle α, measuring the slope of the wire. For $\alpha = o$, there is no reaction, no coupling between x and y. The coupling coefficient is proportional to $\sin \alpha$.

We write down the equations of motion with the terms corresponding to the coupling effect, but restrict ourselves to the case where the angle α is small, so that we may put

$$sin\ ^2\alpha = o, \quad cos\ \alpha = 1.$$

The equations then take the form,

$$(9) \qquad \begin{cases} M' \dfrac{d^2x}{dt^2} + \dfrac{eI}{r^2 l}\left(x - \dfrac{\mu \sin\alpha}{1+\mu}\, y\right) = 0 \\ M \dfrac{d^2y}{dt^2} + \dfrac{eI}{r^2 l\,(1+\mu)}\left(y - \mu \sin\alpha x\right) = 0 \end{cases}$$

Integrating these equations, under certain initial conditions, (e.g. $x = \dfrac{dx}{dt} = \dfrac{dy}{dt} = t = 0$, $y \neq 0$; which means pulling down the spring and setting it free without velocity in the x and y direction), we obtain all the needed information about the time change in amplitudes and the number N of oscillations during one total beat. Resonance occurs if the ratio of the coefficients of x and $\dfrac{d^2x}{dt^2}$ in the first equation equals that of y and $\dfrac{d^2y}{dt^2}$ in the second. The condition of resonance therefore may be written

$$u = \frac{M}{M'}(1+\mu) = 1.$$

u is the same quantity which occurred in the right side of equation (7), and equals the ratio $\omega_x^2 : \omega_y^2$. The number N, expressed in terms of u, is;

$$(10) \qquad N = \frac{1+u}{\sqrt{(1-u)^2 + \dfrac{4\mu^2 \sin^2\alpha}{1+\mu}\, u}}$$

When $u=1$, $\frac{dN}{du}=0$; hence

$$N_{max}=\frac{\sqrt{1+\mu}}{\mu \sin \alpha} \tag{11}$$

N_{max} can be counted very accurately; sin α is obtained by dividing the vertical length of the spring by the length of the wire in the spring. Therefore μ can be calculated very easily from (11). It is instructive also to plot the values of N for different additional weights as a function of u and to compare with equation (10).

UNIVERSITY OF MUNICH.

Spectrophotoelectrical Sensitivity of Argentite (Ag_2S).—The present paper, considered in connection with some previously published data, represents a study of the effect of crystal structure upon photoelectrical sensitivity, as observed in silver sulphide, Ag_2S, in the isometric form argentite, and in the orthorhombic form, acanthite.

Experimental data are given on the effect of temperature, of the intensity of the radiation stimulus, and of mechanical working of the material, upon the spectrophotoelectrical sensitivity of argentite. These observations are then compared with similar data, previously published, on acanthite.

Argentite reacts slightly photoelectrically to radiations of wave lengths 0.5 to 1.1μ, followed by a sharp, fairly symmetrical maximum at 1.35μ. In contrast with this is acanthite, which reacts quite strongly to radiations at 0.5 to 1.0μ followed by an unsymmetrical maximum at 1.35μ. Both minerals react selectively to ultraviolet rays.

At low temperatures, the intrinsic photoelectrical sensitivity of argentite is greatly increased and the maximum shifts to the short wave lengths,—to 1.1μ as compared with 1.2μ for acanthite, under similar conditions.

The photoelectrical reaction of argentite differs from that of acanthite in being free from an induced photonegative polarization.

In argentite, as previously observed in acanthite (also molybdenite, etc.), increasing the intensity of the radiation stimulus produces a more rapid reaction in the long wave lengths than in the short wave lengths, with a consequent shift of the maximum of the photoelectrical sensitivity curve toward the long wave lengths.

Hammering the crystals of argentite and acanthite into thin, pliable, plates lowers the intrinsic photoelectrical sensitivity. The position of the maximum of spectrophotoelectrical sensitivity of the worked samples is less affected by changing the temperature than obtains in the natural crystalline state; and it is practically the same for these two minerals, at low temperatures. From this it appears that aside from

the effect of crystal structure, silver sulphide has a photoelectrical response spectrum characteristic of this substance.

From a comparison of the spectrophotoelectrical reactions of these two crystal forms of silver sulphide under various conditions it appears permissible to conclude that crystal structure has a marked effect upon spectrophotoelectrical sensitivity. [W. W. Coblentz, Bureau of Standards Scientific Paper No. 446.]

Application of Quantum Theory to the Action of Photochemical Sensitizers.—In a mixture of ozone, oxygen, and chlorine, the rate of transformation of ozone into oxygen is accelerated by irradiating the mixture with light which is absorbed only by the chlorine. Measurements were made by Bonhoeffer on the augmentation of the rate of increase in pressure in such a mixture when illuminated by violet light from the mercury arc ($\lambda 406$–$436 m\mu$). This rate is found to be independent of the concentration of ozone molecules over the range (5%–0.1%) of concentrations examined, and proportional to the quantity of light absorbed by the chlorine, (determined by separate tests on the absorption of chlorine-air mixtures of various compositions). The ratio (number of ozone molecules transformed)/(number of quanta absorbed) was determined in seven tests as: 2.33, 1.99, 2.03, 1.84, 2.25, 1.94, 1.9, 1.79—mean value 2.03. This suggests that a chlorine molecule absorbs a quantum and holds it at least until the next collision with an ozone molecule, although this involves (in the mixture with 0.1% ozone) a period of 10^{-7} second during which a thousand collisions with O_2 and Cl_2 molecules may occur. But this quantum must bring about the dissociation of two ozone molecules and their recombination into three oxygen molecules, and apparently there is no satisfactory picture of this action. In a mixture of ozone, oxygen, and bromine vapor the reaction is much more rapid, averaging 30 dissociations per absorbed quantum. The union of CO and Cl_2 is much more rapid yet, thousands of molecular processes occurring per absorbed quantum. [K. F. Bonhoeffer, ZS. f. Physik, *13*, pp. 94–105; 1923.]

KARL K. DARROW

ANOTHER MERCURY VAPOR PUMP

By L. T. Jones

A large variety of designs of mercury vapor pumps have been described during the past few years.[1] Some of the pumps are a little spasmodic in their functioning. When constructed of glass they are usually either quite difficult of construction or are very fragile.

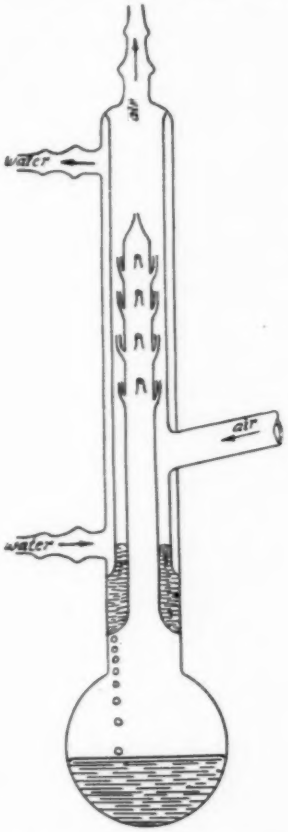

Fig. 1. *Design of mercury pump*

[1] Summary in Gen. Elec. Rev., *23*, p. 605, 672.

The design shown in the accompanying figure embodies all the features necessary for ruggedness with a record of very consistent and satisfactory performance. The glass-blowing difficulties, while too much for the novice, are yet not severe. At first sight the most difficult operation would seem to be that of bringing the air intake out through the water jacket. As a matter of fact this is accomplished quite as easily as an ordinary *T* joint is made. If the diameters of tubing chosen are such that the annular space for water is about 2 mm the outer tube is readily melted down against the inner wall of the water jacket and blowing brings the inner tube right out through the outside wall. It is necessary, of course, to use judgment in the size of the area heated. A considerably more difficult operation is that of leaving the proper size hole through which the condensed mercury returns to the reservoir. This should be of about 1 mm diameter.

The jets through which the mercury vapor escapes from the reservoir are made "cockle burr" fashion, each opening being a half mm or less in diameter. It is advisable to have a large number of such openings in order to have a whole region of vapor moving upward. The number shown in the illustration is sufficient.

The action of the pump is quite like that of other vapor pumps. It must be backed by a vacuum of .1 mm or less. The directions taken by the air and the water for cooling are indicated by the arrows. The vapor rises from the reservoir, escapes through the several openings, carries the gas upward with it and then condenses. The condensed mercury returns to the reservoir through the small hole, as indicated. When the proper supply of heat is applied to the reservoir a 3 cm column of liquid mercury will be maintained above the return opening, effectually sealing the opening.

It is not advisable to attempt to construct the pump of any glass other than Pyrex or G-702-P. To construct, begin at the top and perform the operations in order. The cockle burr should be inserted only immediately before it is sealed into place. Blowing the bulb completes the pump.

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A DEVICE FOR ROTATING SPECTROMETER CRYSTALS

By Paul Kirkpatrick

The method of measuring intensities of X-ray reflections by a slowly rotating crystal, proposed by A. H. Compton, would appear to have established itself as a standard method if one may judge by the number of recent researches employing this means. Accordingly some mechanism for accomplishing this rotation would seem an appropriate adjunct to every Bragg spectrometer. The following described device is directed toward this end. As it has been in regular use in this laboratory for more than a year and has been found very satisfactory it is felt that it will be of interest to other users of the Bragg spectrometer and may prove a slight addition to the usefulness of this fine instrument.

The mechanism comprises a ratchet and sprocket wheel mounted on a common horizontal shaft and situated on a shelf directly on the axis of the spectrometer at a height of about a meter above the instrument. The ratchet may be caused to rotate in an intermittent manner by a pawl actuated by an electromagnet. The resulting rotation of the sprocket is transmitted by a light chain to a lower sprocket integral with the head of the tangent screw provided for imparting slow motion to the spectrometer crystal holder. Reference to Fig. 1 will make the action clear. The pawl A is carried by the link B and the spiral spring shown. Its motion is limited by the two adjustable screws, the lower of which is fitted with a shock-absorbing bit of rubber stopper. There is also a second pawl, C, held against the ratchet wheel by a spring and intended to prevent any tendency of the wheel to rotation in the wrong direction due to friction with A. The member C is slotted for the greater part of its length; A and its spiral spring and the bar D projecting through the slot. The metal parts with the exception of the solenoid and its core are of brass. The base and the large upright member are of wood. In the upper right corner of the figure will be seen a lever provided with a pull-cord for disengaging the pawls from the ratchet as is necessary when backing off the tangent screw preparatory to a setting. This lever disengages C directly and A by pressing against the tension wire connecting A with the solenoid plunger. The rocker and mercury cups depicted in the lower right corner are attached to a

metronome and perform the function of admitting accurately timed current impulses to the electromagnet. The two switches, which should be mounted conveniently to the position of the observer at the spectrometer, permit the use of either one or both contacts. The switches, in combination with the slider of the metronome, give the wide range of from 20 to 208 advances of the crystal per

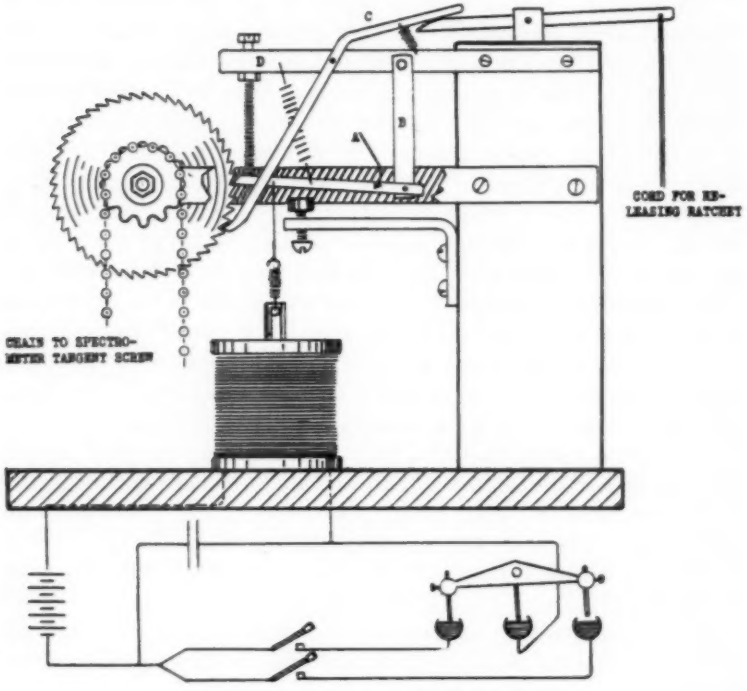

FIG. 1. *Crystal Driving Mechanism*

minute. With our apparatus each advance is of the order of one minute of arc. A six volt storage battery is used as current source. The condenser (of about 1 mf capacity) is necessary to prevent rapid burning away of the dipping contacts.

In the measurement of reflected intensities the position of the ionization chamber is first fixed, the crystal is then placed almost at the threshold of reflection, the state of charge of the electrometer is read, and the rotating mechanism started by closure of the switch. The

crystal is thus turned gradually into its reflecting position and the electrometer may be casually observed until reflection has ceased to occur. The rotating mechanism is then cut out and the final condition of charge of the electrometer noted, the increment of charge being taken as the measure of the reflected intensity. This method of measurement has been fully discussed in various papers, particularly in one by Bragg, James, and Bosanquet.[1]

It will be found however that the usefulness of the rotating device is not limited to its connection with the intensity measurements. It forms a most convenient aid in locating spectral lines, absorption discontinuities, or other special spectral features. In picking up a reflection from the general spectrum one is enabled to sit at the electrometer and wait for the reflection to come by instead of going through a tedious series of alternate settings and readings. When a reflection is located the crystal may be adjusted to peak reflection by first noting the total charge developed as the crystal is driven through the position of reflection and then repeating the run as far as the point at which half of the previously recorded charge has been accumulated.

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[1] Bragg, James, and Bosanquet Phil. Mag., *42*, p. 1, 1921.

OPTICAL SOCIETY OF AMERICA

EIGHTH ANNUAL MEETING,

Cleveland, October 25-27th, 1923

PRELIMINARY ANNOUNCEMENT

The Eighth Annual Meeting of the Optical Society of America will be held at Cleveland, Ohio, Thursday, Friday and Saturday, October 25th, 26th, 27th, 1923.

The regular sessions for the reading of papers will be open to all interested persons.

Members and others desiring to communicate results of optical research are invited to submit titles of papers for the program to the Secretary any time before September 10th. *Titles received after that date cannot be included in the program. There will be no "supplementary program."* Each title must be accompanied by an abstract ready for publication. These abstracts will be printed in the program and in the minutes of the meeting. The purpose of the abstract is to give: (1) a more definite description of the *nature* and *scope* of the paper than can be conveyed in the title, (2) the essential results insofar as may be possible in the limited space allowed. It is hoped that the advance publication of these abstracts will prepare those attending the meeting to consider the papers more intelligently and with much greater interest. Printed forms on which to submit titles and abstracts may be obtained on application to the Secretary. Inasmuch as their use will greatly simplify and expedite the work of editing and printing the program, the Secretary earnestly requests that these forms be used in submitting abstracts.

Attention is invited to the following resolution adopted by the Council at its Annual Meeting, October, 1922:—"*Resolved:* That it is the sense of the Council that ordinarily, members should not present without invitation, papers which have already been published."

No title will be printed to be presented "by title." Titles should not be submitted unless the author has a bona fide intention to actually present the paper orally or have it presented by some one else.

IRWIN G. PRIEST, *Secretary*,
Optical Society of America,
Bureau of Standards,
Washington, D. C.

Committee on Papers
for Cleveland Meeting
HERBERT E. IVES, *Chairman*
W. E. FORSYTHE
H. G. GALE
IRWIN G. PRIEST
CHARLES SHEARD

DISAPPEARANCE OF THE FILAMENT AND DIFFRACTION EFFECTS IN IMPROVED FORMS OF AN OPTICAL PYROMETER[1]

By C. O. Fairchild and W. H. Hoover

SYNOPSIS

The precision, accuracy and wide range of usefulness of the disappearing-filament optical pyrometer depend on perfect disappearance of the filament with high resolving power and magnification by the eyepiece. Older forms have not accomplished this. Nondisappearance of the filament is caused by diffraction, and if the filament is a wire, by reflection at its edges. Perfect disappearance of a flat filament with sharp edges is obtained by using an entrance aperture sufficiently large compared to the exit aperture. A round filament is made to disappear by decreasing the entrance aperture until diffraction and reflection nearly balance, in their effects at the filament.

An extensive study of diffraction revealed that diffraction by an obstacle in an image plane is a special case which has not been previously recognized as such. It is shown that a new law of diffraction is consistent with theory and experiment, viz., *Of light incident on an obstacle in an image plane, only that component is diffracted which represents the diffracted rays radiating from the edge of the aperture.* Thus with the flat filament there is no visible effect of diffraction if the aperture of the observing eyepiece (exit aperture) is far outside the shadow of the entrance aperture.

Precise measurement shows that when the filament is made properly to disappear, the ratio of brightness of filament and image when an apparent match is obtained, is unity.

A laboratory form of the pyrometer is described, which has been used with success as a tele-pyrometer and as a micropyrometer. Two special improved forms of micropyrometer, or microphotometer are described in principle.

Introduction

The disappearing-filament optical pyrometer consists simply of a telescope or microscope in a focal plane of which is placed an electric

[1] Published by permission of the Director of Bureau of Standards.

lamp with a small filament. Temperature of a glowing object is measured by matching the brightness of the filament with that of the image of the object, projected in the plane of the filament.

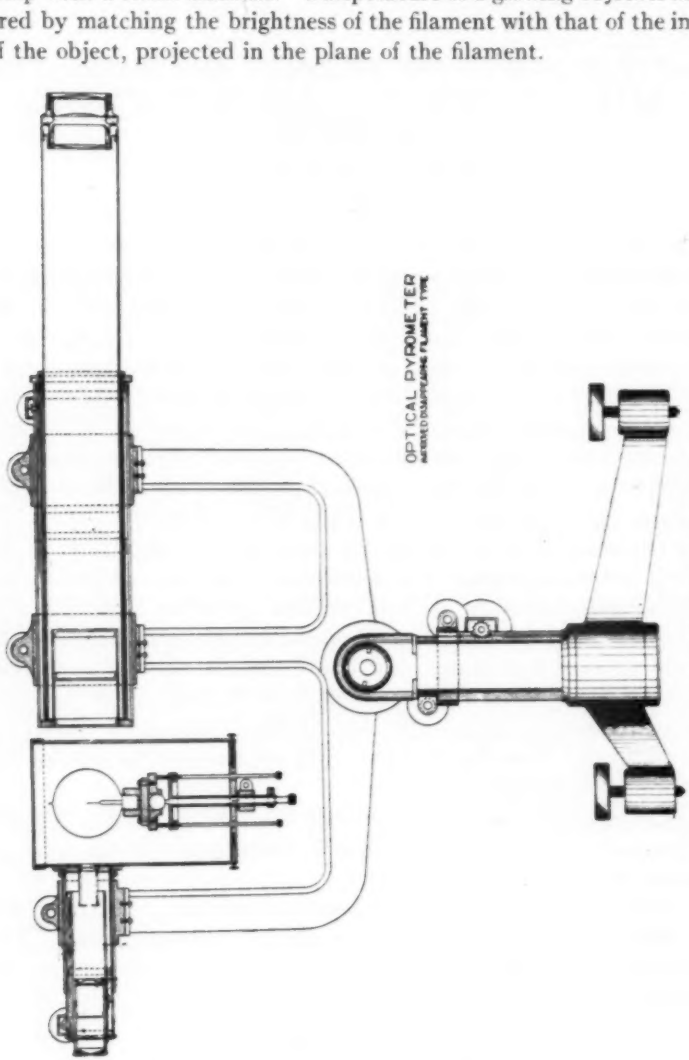

FIG. 1. *View of Optical Pyrometer.*

The form of the instrument now being used by the writers at the Bureau of Standards is shown in Fig. 1. Certain details of construction have been developed from the study recorded here. The method

of support of the telescope leaving space for interposing a sector disk was copied from a design by Forsythe. Fig. 1 is placed here to aid in discussions following but a detailed description is found in later sections.

This type of pyrometer excels other types in ease of manipulation and in precision at high temperatures. Although the relation between the optical scale of temperature and the thermodynamic scale is not as well established as that between the gas thermometer or the total radiation pyrometer and the thermodynamic scale, the first is by far the more useful. The uncertainty of the optical scale is not great, and the scale has been found to agree very well with other scales. However, to take full advantage of this form of pyrometer it must be properly designed as an optical instrument, with attention to both geometrical and physical optics. We have attempted to do this—with apparent success. Previous designs have been seriously at fault in major details, and thus usefulness of the laboratory forms of the instrument has been considerably restricted. Our studies will result, we hope, in much greater confidence in the optical pyrometer as an instrument of precision. In its improved form it is probably the most precise of any form of photometer.

The precision attainable depends on perfect disappearance of the pyrometer filament when the current through the filament is adjusted until the brightness of image and filament are apparently equal, or matched. Furthermore such disappearance must take place with a sufficiently high resolving power and magnification by the eyepiece so that the border lines between filament and image or background may appear quite sharply defined and be easily visible when a slight inequality of brightness exists.

The accuracy of the instrument is limited by the constancy of the relation between the current through the pyrometer filament and the brightness of the source sighted upon. Since the brightness of the image of the source bears a constant relation to that of the source, then the ratio of brightness of filament and image, when apparently matched must be invariable.

Worthing and Forsythe[2] after an extensive study concluded that in general this ratio was not unity, and they described precautions for keeping it constant. More than this they described special conditions under which disappearance of the filament was possible, and concluded that the appearance (non-disappearance) of the filament resulted

[2] Phys. Rev., Series 2, *3*, p. 163; Sept., 1914.

entirely from diffraction by the filament. Likewise the variation in the ratio of brightness of filament and image was assigned solely to the effect of diffraction. The published work of these writers has stood for eight years as a proof of serious limitations to the usefulness of this optical pyrometer. We will take exception to the conclusions of Worthing and Forsythe in later paragraphs, not forgetting however that they first showed that inattention to the effects of diffraction might lead to serious errors in temperature measurements.

Our studies have shown that true equality of brightness of filament and image is readily attainable, and have revealed a simple test of this equality. Perfect disappearance has been obtained with very high resolving power and any desirable magnification.

Appearance of the Pyrometer Filament

The original type of this instrument, the Morse or Holborn-Kurlbaum pyrometer, is a low power telescope, in which the lamp filament in the image plane is but slightly magnified by the eyepiece. The magnification is ordinarily about two or three. Although the lamp filament is very small, under careful observation, when a brightness match of filament and image is attempted, the filament will not disappear but will be bordered by faint bright lines. It is shown below that these are caused by reflection of light incident on the round filament. No effect of diffraction is visible.

Few attempts have been made to develop or perfect the ordinary form and obtain an instrument of precision. Worthing and Forsythe first modified the instrument to measure temperatures of lamp filaments. Finding some peculiar errors of measurement in preliminary experiments they were led to the investigation mentioned above.

Two ways in which errors may arise are to be mentioned. First, if the filament will not disappear when the best apparant match is obtained, but remains visible with bright or dark edges, it is found that two observers will not judge the best brightness alike, nor will one observer obtain precise settings without very careful training of his memory of the *appearance* of the filament when his setting is made. The error caused by non-disappearance is never more than a few degrees. Second, if the resolving power of the eyepiece is fixed at a very low value to make disappearance possible, Worthing and Forsythe found that when an apparent brightness match is made, the ratio of real brightness of the filament to that of the image may vary from less than unity to nearly 1.7, representing a great error in temperature.

They concluded that this ratio depended on the diameter of the filament and the value of the entrance aperture, and that in general the ratio was not unity, but approached an asymptotic value of about 0.9 for large apertures. Irrespective of what this ratio might be they further concluded that the aperture stops must be fixed, and selected very small apertures as the best for good disappearance. In a particular case: "For a 0.050 mm tungsten filament using red light, a very satisfactory value for the entrance cone angle was found to be 0.05 radian." Their exit aperture value is not stated but was probably about 0.015 radian. This small aperture of the eyepiece corresponds to a resolving power of about 0.045 mm or nearly the diameter of the filament.

Only with such low resolving power of the eyepiece is it possible to encounter great error in temperature or brightness measurements, for in this case an optical effect at the edges of the filament may be averaged with the true brightness of the filament. Thus an apparent match could be obtained with an effect of diffraction which destroyed real or true equality of brightness of filament and image. The filament could, in truth, be made to disappear.

Now if the resolving power of the eyepiece be increased to reveal the effect at each edge of the filament, if such effect be existent, then perfect disappearance of the filament with this eyepiece is only possible when the effect is absent. In such case then, as we will prove later, with perfect disappearance there will be actual equality of brightness of image and filament.

To find what conditions make disappearance possible with the higher resolving power of the eyepiece it was necessary to study the appearance of the filament under all the conditions which prevent disappearance. Proper interpretation of observations and results, and the creation of confidence in their validity led to a rather extensive study of diffraction and a long and tedious series of experiments. A description of the work in chronological order would be as tedious and would reveal some mistakes and erroneous conclusions. We were not the first to be confused by the effect of lens imperfection and reflection from round filaments. Avoiding such confusion for the moment we will briefly describe the appearance of a flat filament as viewed with various aperture stops.[3]

[3] Flat, sharp-edged filaments were made by grinding and polishing round filaments to a semicircular cross-section.

With an exit aperture of 0.08 radian (or smaller) and an entrance aperture of about 0.20 radian (or larger) the matched filament (flat) disappears completely.* Fig. 2B. As the entrance aperture is gradually decreased a barely visible dark line appears just outside each edge of the filament. Fig. 2D. This dark line increases in intensity and apparent breadth until the entrance aperture equals the exit aperture. Further reduction of the entrance aperture brings to view a bright line outside each dark line. Fig. 2F.

With a large entrance aperture, if the exit be reduced the filament remains invisible against the background but with the effect (poor

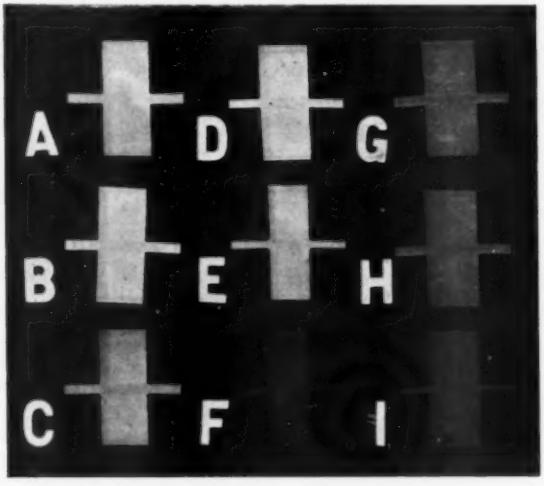

FIG. 2. *Appearance and Disappearance of Pyrometer Filament.*

resolution) shown in Fig. 2E. The exit aperture is here 0.02 radian. Reduction of the entrance aperture as before, results in a similar series of effects, the lines appearing broadened. With an exit aperture of 0.02 radian, however, the entrance aperture may be reduced to about 0.07 radian before the dark lines are visible. This is illustrated in the table on page 549.

Exit apertures as large as 0.10 radian require a filament quite free from flaws, because of the increase in definition.

* Fig. 2, A, B, C are not successful half-tone reproductions. The "match" may appear in the wrong figure, or no match at all may appear in any case shown.

The appearance of a round filament with various apertures differs from that of a flat filament because of reflection by the filament of converging incident light. Thus at each edge of a round filament there may be a bright band of reflected light Fig. 2 G, or a dark band from diffraction, or both. Fig. 2H. The former will appear inside the edge of the filament and the latter outside.

The bright band of reflected light will increase in intensity with increase of either entrance or exit apertures. The area of the surface of the filament, from which reflection into the exit orifice occurs, is proportional to the diameter of the filament and the sum of exit and

TABLE 1. *Minimum Entrance Aperture Required for Various Exit Apertures. Flat Filament*

Exit Aperture Radians	Difference	Entrance Aperture Radians Minimum
0.02	.05	0.07
0.03	.07	0.10
0.04	.09	0.13
0.06	.11	0.17
0.08	.13	0.21
0.10	.13	0.23
0.12	.13	0.25

entrance angles. The intensity of the reflected light will vary in somewhat the same proportion. The dark band of diffraction will appear nearly as with the flat filaments, but the combined effect of both bands is of greater interest.

An exit aperture of 0.06 radian is barely sufficient to resolve the two lines, one of reflection, and one of diffraction by a 0.050 mm filament. To render them sufficiently plain to illustrate Fig. 2H, the exit and entrance angles were, respectively, 0.08 and 0.10 radians, and a 0.10 mm filament was used to further increase the contrast and visibility.

The following changes take place in the appearance of a round filament (about 0.05 mm diameter) as the apertures are altered.

With entrance aperture of 0.16 radian, an exit of 0.10 radian reveals dark and bright bands. Increasing the entrance results in eventual disappearance of the dark and increased brilliance of the bright band. It may be mentioned here that imperfect resolution may cause the dark band to disappear as the entrance is increased at a smaller difference of entrance and exit than called for by a flat filament, on account of the contiguous bright band. This will be more apparent

presently. Now, as the entrance is decreased from 0.16 radian, the dark band becomes more apparent, the bright one less. But with this exit (0.10) the filament will not disappear with any entrance angle whatever.

No marked change in the above takes place for smaller exit apertures, until the exit is reduced to 0.06 when with a certain entrance aperture the filament nearly disappears. If the exit be further reduced to 0.03 or 0.04 radian, the bright band alone is visible with an entrance of 0.16 radian. Now if the entrance is decreased, the lesser resolving power of the eyepiece causes the bright and dark bands to merge and disappear against the background if they are not too different in intensity. Fig. 2I. Thus for entrance angles from 0.08 to 0.13 and exit 0.03 or 0.04, the .050 mm filament disappears. The relative brightness of image and filament remains constant (equal) when apparently matched, over this range of apertures giving perfect disappearance.

Smaller exit values (than 0.04) permit a wider useful range of entrance apertures. With an exit 0.02, the entrance may vary from 0.06 to 0.16 without preventing disappearance. An exit of 0.01 radian permits the use of any entrance over 0.04 radian, but such a small exit is inadvisable, because definition is then so poor. There will be, however, no large error in temperature measurement, so long as the entrance aperture is kept greater than 0.04 radian.

The largest round filament which can be made to disappear without unduly low magnification or resolving power has a diameter of about 0.100 mm or 4 mils. The best apertures for this large filament are exit 0.02 and entrance 0.06 or 0.07 radian. This large filament is seldom used, however. Difficulty is encountered unless the aperture stops be fixed, and well aligned.

It will be well to restate the case of the 4 mil filament.

With an exit aperture which will resolve the edge effects fairly, such as 0.02 radian, the two conditions of sufficiently small entrance aperture to avoid visible reflection, and large enough to avoid visible diffraction are barely met.

In Table 2 are given the various aperture values to be used with round filaments of the diameters given.

Relative Advantages of Round and Flat Filaments

The flat filament is particularly useful in forcing precision of measurements to a maximum and in measuring lower temperatures, i.e., down to the melting point of aluminum or of antimony.

However, a large difference in the apertures of exit and entrance stops are required for a flat filament. The relation between entrance aperture, magnification of source, and working distance, may require an objective lens of unattainable aperture ratio, unless the round filament be used. (Assuming that we do not resort to very low resolving power or magnification of the pyrometer filament or both.) The relation will be definitely stated in a later section. Such a contingency will be rare, and the special advantage of the round filament lies only in the nuisance of preparing the flat one.

TABLE 2.[4] *Round Filament. Diam. 0.04 to 0.06 mm*

Exit Aperture Radians	Entrance Aperture Radians
.04	.08 to .13
.02	.06 to .16
.01	.04 and larger

Round Filament. Diam. 0.1 mm

Exit Aperture Radians	Entrance Aperture Radians
.02	.055 to .07
.01	.04 and larger

IMAGE BRIGHTNESS

We have stated in previous paragraphs that perfect disappearance of the pyrometer filament, with high resolving power and sufficient magnification, gives true equality of brightness of image and filament when they are apparently matched. In the absence of any resultant optical effect at the edges an experimental proof of this equality would hardly seem necessary. However, Worthing and Forsythe[1] have obtained results which we have not been able to verify.

There is only one method, in principle, of experimental determination of the ratio of brightness of filament and image. This is done by using one lamp both as a pyrometer lamp and as a source, or in some way exchanging filament and image. The method of Worthing and For-

[4] The values in this table are for tungsten filaments. Carbon filaments are no longer used and were not studied. A lower reflecting power, (25% of that of tungsten) would require the use of a larger entrance aperture for a given exit aperture and filament diameter. The bright line of reflection at the edge would be balanced by a less dark line of diffraction, which is given by a larger difference between entrance and exit apertures.

sythe was as follows: Three lamps numbered 1, 2 and 3 were used. Lamp No. 1 as a pyrometer lamp was sighted upon No. 2 as a source, and image and filament matched. Then No. 3 in the pyrometer was used to determine the relative brightness of 1 and 2 at the currents obtained in the first case. The manner of applying corrections for transmission of lamp globes and pyrometer objective was not described.

Our first attempt to estimate apparent image brightness resulted in an error of 2%, the next decreased the error to 1% and finally very high precision was obtained. The first errors were occasioned by poor lamp globes and a lens of low transmission. The first lens used as a front objective was a Zeiss-Tessar F4.5 with a transmission of 71%, for red light. Later, lamp globes with windows of optical glass were made, and three lenses of high transmission (88.0%) were obtained. These lenses were computed by the optical division of the Bureau of Standards, and were made of selected pieces of crown and flint glass made at the Bureau. Two of these lenses were used in the measurements of image brightness. The transmission of this cemented lens computed from the indices of refraction is 89.7%. The lower value obtained experimentally (1.7% less) is caused by a reflecting power greater than the theoretical value. Such a difference has been noted many times, and various authors have described surface effects of such a magnitude.

To obtain high precision in an estimate of image brightness the following series of measurements were made.

Although two lenses, for pyrometer objectives, of identical construction, were available, their transmissions were measured and as to be expected were found equal within experimental error—the results differing by less than 0.1%. Two pyrometer lamps with flat optical glass windows were used.

Lamp No. F 10 was carefully calibrated at the melting points of copper and gold, by sighting into a black body immersed in the metal. Lamp F12 was calibrated in the same manner at the copper point. Then, maintaining the current in F12 at the value for the copper point, it was used as a source and matched with F 10 in the pyrometer. The same window of each lamp was always kept toward the observer. The brightness of the image of F12 was less than that of F12 by a factor to be measured.

Let J be the brightness of the black body at the copper point, and J_{10} the brightness of F10 matched against the image of the opening of the black body. Let T_1 and T_2 be transmissions of rear and front windows of F10, and T_3 and T_4 corresponding transmissions of F12.

Let T_L be the transmission of either of the two identical lenses used. Then, if K is the ratio of brightness of filament and image, when apparently matched

$$J_{10} = K\ J\ T_2\ T_L \tag{1}$$

Similarly for F12 at the copper point

$$J_{12} = K'\ J\ T_4\ T_L \tag{2}$$

When F10 is now sighted upon F12 held at the brightness J_{12} as described above,

$$J_{10}' = K''J_{12}\ T_2\ T_L\ T_3 \tag{3}$$

Lamp No. F10 was finally sighted upon the black body at brightness J with the duplicate lens and F12 interposed between pyrometer and source. Within experimental error the current through F10 was found to be the same as that required when sighting upon F12, corresponding to the brightness J_{10}'.

Hence

$$J_{10}' = K''\ J\ T_2\ T_3\ T_4\ T_L^2 \tag{4}$$

From (3) and (4)

$$J_{12}\ T_2\ T_3\ T_L = J\ T_2\ T_3\ T_4\ T_L^2 \tag{5}$$

or

$$J_{12} = J\ T_4\ T_L$$

and from (2) $K' = 1$.

It was proved that the ratio of brightness K' of the filament of F12 and image was unity.

The actual results obtained for equations (3) and (4) were, in terms of the current through F10:

0.11382 ampere, sighting on black body.
0.11378 ampere, sighting on F12.

The difference 0.00004 ampere is equivalent to 0.2°C or 0.3% in brightness for red light. The transmission of both windows of F12 was determined separately and this, combined with the previous determination of the transmission of the duplicate lens, gave a final mean of all readings by two observers which was equivalent to an error of 0.05% in the apparent image brightness.

The lamps F10 and F12 have filaments which are, respectively 0.037 mm and 0.10 mm in diameter. The filaments are in round wire form and a question arises as to why no effect of the deviation from Lambert's cosine law, of the emission from the filament of F12 was detected. This deviation is effective near the edges of the filament and is accounted for, by the selection of such apertures that the dark band of diffraction, the bright band of reflection and the variable brightness

near the edge at small angles of emission are balanced and the exit aperture is selected to just prevent resolution of the separate effects.

Worthing and Forsythe found that with large apertures the ratio K seemed to approach a value of 90 or 91%. They obtained this result with an entrance aperture of about 0.10 radian, but failed to record the value of the exit aperture used. An exit was used, such that reflection predominated over diffraction. It is readily apparent that K will continue to decrease as the apertures are increased in value.

One of the conclusions of Worthing and Forsythe was that the temperature of a lamp filament should be measured by using it as a source or background and not as a pyrometer filament. Otherwise, an error would arise caused by diffraction. We have definitely and conclusively shown that the temperature of a small filament can be accurately measured by placing it as a pyrometer filament. Moreover, we will show in the succeeding section on geometrical optics, that the small filament is more easily tested by the latter method.

This section is concluded by stating one important working precaution in the use of the disappearing-filament pyrometer:

The magnification and resolving power of the eyepiece should be sufficient to reveal any optical effect at the edges of the pyrometer filament, and the aperture stops should be selected to provide perfect disappearance with this eyepiece. When the stops are properly chosen the magnification by the eyepiece can be changed at will, this being an excellent means of adjusting the brightness to the best value for accurate photometric work.

Principles of Geometrical Design Which Fulfill the Above Requirements

We have stated in the introduction that the disappearing-filament optical pyrometer is not described in any treatise on optics. To formulate some general principles and enumerate certain conditions which will render the geometry of the instrument consistent with the physical requirements seems to be demanded. These requirements are determined by the theory of diffraction, the theory of stops and some simple photometric considerations.

Disappearance of the pyrometer filament is obtained when the entrance and exit apertures are selected within certain limits. It happens that with small round (wire) filaments the best exit aperture is 0.04 of a radian. This small angle necessitates a rather small physical stop or limiting diaphragm, if the length of the instrument is to be

kept at a convenient magnitude. So far as diffraction and definition are involved the actual size of this stop is immaterial, providing only that it is large compared to the wave length of light. That is diffraction by the filament, and resolving power (or definition of the field) are determined by the angular aperture only. Hence, stops may be placed in the instrument at any point, without considering their actual size,—provided that the position will not unduly decrease the field of view.

The simplest way to diaphragm or limit the apertures—the method used by Worthing and Forsythe—is shown in Fig. 3 in which AB is the exit diaphragm (entrance of observing microscope) and CD is the entrance diaphragm or objective aperture.

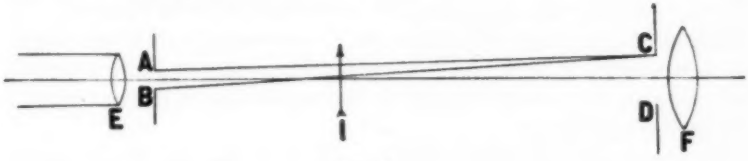

FIG. 3.

The field of view of the external source is bounded by the line AC, and the field of undiminished aperture (uniform brightness) by BC. The microscope objective E is behind AB. In this arrangement, unless the pyrometer is very long, 2 meters or more, the field of view is small and not uniformly bright, but appears with a bright center. The entrance pupil, that is the image of AB on the object side, is usually smaller than AB and varies in size and position with movement of the objective lens F.

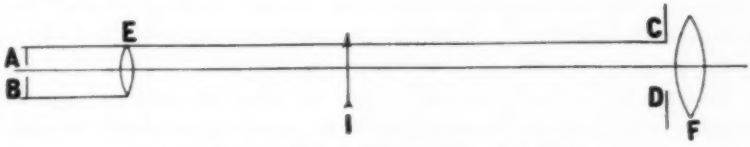

FIG. 4. *Improved Method of Diaphragms. See also Fig. 1.*

To improve the condition of small field, and to take full advantage of the use of a small exit aperture, we have placed the exit stop AB behind the microscope objective E, *at the principal focus of* E, as shown in Fig. 4. The field of view is now bounded approximately by the line EC, since AB is small. The field of undiminished aperture is practically the entire field. If CD is not smaller than the objective E the

fields of view of both image and pyrometer filament are the same, and are sufficient to much more than fill the Huygens eyepiece. The entrance pupil (an image of AB) is now at the principal focus of the front objective on the object side; and its diameter is $\frac{F}{f} \cdot AB$ where F and f are the focal lengths of front and rear objectives. Moreover the entrance pupil is fixed in position with reference to the front objective. The entrance window (image of the diaphragm of the eyepiece) is in the object plane, and the field is sharply outlined.

The exit pupil is of course rather small, depending on the eyepiece, but is apt to be too far from the eye lens unless the focal length of the microscope objective is 50 mm or more. A convenient plan is to use a 50 mm objective at a magnification of two and a $\times$ 10 eyepiece. To measure temperatures lower than 800°C conveniently, a lower magnification should be used. If the exit pupil recedes too far an eye rest may be attached to hold the eye at the required distance, in excess of the standard 15 mm.

However, unless a large source is viewed so that the whole field of view is illuminated the exact position of the exit pupil is not important.

Ordinarily the exit window is not limited by the pupil of the eye. On occasion the pyrometer may be used to sight through a small window into a furnace. Not only the entrance aperture but the exit window may be altered.

It can be easily demonstrated that the angular aperture of the observing microscope, that is the exit angle is $\frac{D}{f}$ where D is the diameter of the stop AB, Fig. 4, and f is the focal length of the microscope objective. Thus the exit aperture is independent of the position of the eyepiece although the stop AB is not fixed with reference to the filament, as in Fig. 3, but moves with the eyepiece.

The space between the two objectives being telecentric multiple stops to cut down stray reflection, should be all of the same size and slightly larger than the rear objective.

The principal rays through both objectives pass through their principal foci, and in this manner the action of the lenses is improved, over that accompanying any other position of the stop AB. This will be the case, at least, for all ordinary objectives. It is not meant that the position of the stop aids in the riddance of visible diffraction at the filament, but only that it improves the image as seen through the whole instrument.

The small exit aperture permits the use of ordinary lenses in the eyepiece microscope. A simple achromatic 50 mm. objective and a Huygens eyepiece are very satisfactory.

The front objective, on the other hand should be selected with care, because a large entrance angle is required. The following simple relations between entrance aperture, magnification, working distance, and aperture ratio will suffice to determine somewhat the type of front objective.

Let w = working distance
m = magnification by front objective
f = focal length of front objective
d = diameter of front objective
φ = entrance angle

For thin lens:

$$w = \frac{m+1}{m} \cdot f$$

$$\varphi = \frac{d}{(m+1)f} \text{ or } f = \frac{d}{(m+1)\varphi} \text{ or } m = \frac{d}{\varphi f} - 1$$

$$\frac{mw}{f} = \frac{d}{\varphi f} \qquad \text{or } d = mw\varphi \qquad \text{or } w = \frac{d}{m\varphi}$$

If φ is selected then d and f may be determined from m and w or vice versa.

Example:

An F3 lens is available.

The entrance aperture φ must be at least 0.07 radian.

$d = 45$ mm $\quad f = 135$ mm

Then m cannot be greater than 3.75 and w is about 170 mm.

This F3 lens is the one now in use in the instrument illustrated in Fig. 1. With it the instrument can be used as a micropyrometer with a working distance of over 170 mm and a total magnification of 90 or more. We have used it to determine the melting point of as little as 0.003 mg of a refractory substance.

This large instrument which may be used either as tele-or micro-pyrometer is necessary in the latter case when a long working distance is required, as when sighting into a very hot furnace. For special and restricted use, the following designs are preferable.

I. Low power micropyrometer for sighting on hot sources >0.1 mm diameter.

Front objective Tessar 1C, F3.5, efl 50 mm
Working distance 60 mm
Magnification <3
Rear objective 48 mm achromat N.A. .08
Huygenian eyepiece $\times 10$.
Physical stop diameter $48 \times .02 = 0.96$ mm.

To obtain higher precision at lower magnification the exit angle is increased to 0.04 radian, the entrance angle to 0.12 radian and the magnification by the front objective is reduced to <1.4. The lamp filament should not be in either case, larger than 0.05 mm (2 mils). This form is suited to the measurement of temperatures of lamp filaments not less than 4 mils in diameter. We have shown that filaments smaller than this may be used as pyrometer filaments to measure their temperature, contrary to the contention of Worthing and Forsythe that this should not be done. The working distance of 60 mm can be increased to 100 or more when sighting on large lamps with heavy filaments.

II. Microphotometer of small working distance, for measuring transmission of small objects such as spectral lines on photographic negatives.

Front objective 16 mm achromat. N.A. 0.25
Working distance 7 mm
Magnification <6
Eyepiece same as I.

There is no advantage in a higher magnification. An eyepiece magnification of 10, giving a total of 60 will reveal the grain of the negative and make it difficult to obtain a photometric setting.

The micropyrometer, or, as it is sometimes known, microphotometer, when used to measure the "specular" density of lines on a spectrogram, requires a condenser or a lens for giving parallel light. A large brilliant source is necessary if lines of low transmission are measured. The source must be fairly constant. A good method is to use a magnesia-lined light-box as in photometric measurements. Particular care must be taken *that the field (in the plane of the spectrogram) of undiminished aperture is large*. The problem is the same as that of illumination in microscopy. Large aperture of the condenser is unavoidable.

Fault is found in the use of a stock objective at a lower magnification than it is designed for. Use in this manner is only possible with the small stop and particularly when the stop is placed as described above. The Tessar 1c, F 3.5 is not very good at 60 mm object distance.

We will not attempt to better the problem of the front objective until we have the results of the optical division of the Bureau of Standards, on a pyrometer lens computed for our purpose.

Alteration of the industrial form[5] of optical pyrometer:

This instrument, which has been developed to afford necessary precision and accuracy, at no sacrifice of easy manipulation in any industrial application, can easily be altered for special laboratory uses.

In a previous paragraph the appearance of the filament in this instrument was described as with bright bands of reflection at each edge of the filament. These bright bands do not detract from the ordinary usefulness of the pyrometer, because they are only faintly visible. They cause some annoyance and lack of precision in laboratory work such as melting-point determinations or pyrometer calibrations. They can be eliminated simply by putting a limiting diaphragm in

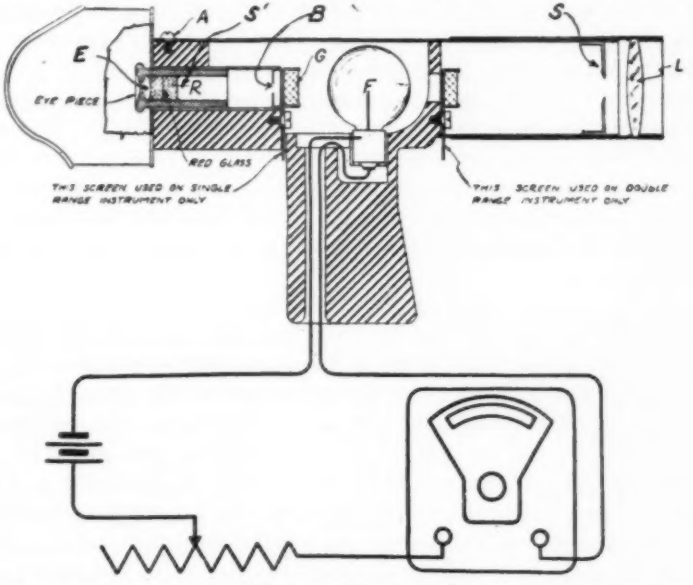

FIG. 4A. *Optical system and electrical circuit of the industrial form of optical pyrometer, changed by addition of the stop S.*

the front end of the telescope tube, at S in Fig. 4A. The diameter of this diaphragm should be such as to subtend an angle of about 0.08 radian. If this does not result in fairly good disappearance, there is

[5] Made by The Leeds & Northrup Co., Philadelphia.

either (1) poor alignment, (2) bad lamp globe or (3) poor filament. A non-cylindrical or split filament will, of course, increase the difficulty in obtaining disappearance.

The exit angle of this instrument is about 0.06 radian, and disappearance of the filament (which is usually about 3 mils in diameter) is only possible with this exit at the low magnification used.

With this low magnification it is difficult to obtain the most precise setting. The eye is kept focused upon the minute filament with considerable effort. To increase magnification of the filament, a 50 mm lens may be substituted for the lens found. This is mounted 50 mm behind the filament, at B in Fig. 4A and a limiting stop is mounted at S′ 50 mm behind the lens. The best diameter for this stop is 1 mm for the 3 mil filament. The entrance aperture, S, must be reduced in this case to about 0.07 radian.

The result of these changes in the previous paragraphs is a reduced field of view and brightness of image. The latter result prevents the accurate measurement of temperatures below 750°C.

We have assumed that for precise laboratory measurements, proper precautions are taken to measure the current through the filament with precision. A potentiometer is necessary. A rheostat with finer adjustment than ordinarily provided, is also necessary.

In its ordinary form, the precision of the industrial instrument is limited by the length of the rheostat and of the scale of the ammeter.

DIFFRACTION

Grimaldi in the 17th century first observed and described diffraction, but since then little attention has been given the special case in which the obstacle in the path of the light is in an image plane. Furthermore, no investigator of diffraction as such has thought to use an obstacle which could be heated to incandescence and matched in brightness with the incident light. So a study of diffraction and its effects, as it occurs in the optical pyrometer has considerable interest. Previous studies in diffraction have all been limited to the use of a point source, the light from which is rendered parallel or left divergent. In the present problems, a finite image is formed and partly obstructed by the filament of the pyrometer lamp. Diffraction, or the illumination of the shadow of the filament occurs. Is this an inherent fault of the pyrometer? The answer is no. In fact, an advantage was found in that diffraction made possible avoidance of the effect of reflection by round filaments.

To show experimentally that a filament can be made to disappear was a relatively simple problem. However, it remains to be shown why the loss of light by diffraction, becomes neglibly small when certain lens apertures are used. Why is the first visible effect of diffraction when the entrance aperture is decreased, a single dark line at each edge of the filament? Can this fact be associated with other observed effects of diffraction?

A complete mathematical solution of the problem would involve expressions too complex to use in the necessary numerical calculations and in fact, would lead us far afield in mathematical exercise. Some of the problems suggested in following paragraphs, may be useful suggestions to present day students of mathematics, interested in optics. As a problem in diffraction we have (See Fig. 5) converging light falling on the straight-edged obstacle E which is incandescent and matched in brightness with the image. A second image is formed on the retina, or in the eyepiece of the observing telescope, and an expression (which may be in two dimensions) might be developed to show the variation in intensity across the second image at right angles to the edge of E.

FIG. 5.

With certain apertures we would find this intensity constant for central parts of the field with the exception of a sharp drop at the edge of the image of E. We finally would desire to know the variation in the intensity of this drop (i.e. the visibility of the dark line) with change of apertures. The expression for the diffraction image of a line source is itself a cumbersome series of Bessel functions, which becomes unmanagable for large apertures. As the intensity of the second image approaches uniformity with the required change of aperture the series integrals would approach in value a single term expressing the intensity given by geometrical optics. We may do well to avoid such a procedure.

A brief summary of the field of investigation covered by mathematicians who have studied problems in diffraction of more or less direct bearing on our problem may be of some interest.

Lommel, in 1884, developed the mathematical treatment of the diffraction image of a point source, as formed by a circular aperture or

ideal lens. This work is referred to extensively by Lord Rayleigh, on pin-hole photography and in his article on the diffraction of light in the Encyclopedia Brittanica, 11th edition. Gray and Matthews in their Treatise on Bessel Functions have made much of Lommel's work readily available. This treatise has been indispensable in numerical calculations and in the development of a few simple relations bearing directly on the problem in hand.

Sommerfeld in 1895 gave a rigorous mathematical treatment of the diffraction by a semi-infinite screen. His results have been simplified and extended by Lamb, Whipple, Hargreaves, Mitra, Banerji and others. The results of these workers have enabled us to explain all of the phenomena we have observed, and have suggested new experiments which were instructive.

Brush has shown that diffraction by cylinders of small radius (up to 1 mm) differs but little from that by a straight edge.

Banerji has extended Sommerfeld's problem to explain the radiation from the edge of a diffracting aperture and Banerji narrowly missed experiments and observations which we found essential.

The flow of energy in a diffraction field was investigated by Chinmayam, who found that in the diffraction field due to a straight edge, the mean direction of flow is not altered by the edge, but that energy "crinkles" along the mean lines, with an amplitude of the "crinkle" which decreases with increasing distance from the edge, finally vanishing.

Polarization of diffracted rays was first shown experimentally by Gouy, who observed the light diffracted into the shadow. Sommerfeld's treatment shows that light diffracted out of the shadow is polarized to the same extent. Also, at small angles of diffraction polarization is negligible, and in the optical pyrometer would only be noticeable with widest apertures.

A diffraction treatment of an actual lens was given by Silberstein[6] in 1918, in whose paper is a reference to K. Strehl's investigation of diffraction by lenses, including the effects of spherical abberation, astigmatism and coma. According to Silberstein who for the first time gave concrete numerical data for a lens, Strehl's series developments are too complicated for numerical calculations. Silberstein's work is limited to the image of a point source or an incident plane wave.

[6] Phil. Mag. *35*, p. 30; 1918.

In what follows it will suit our purpose to emphasize certain fundamental aspects of diffraction:

1. Diffraction may be described as the distortion of a wave front by its partial obstruction.

2. Illumination of the shadow means a *loss of light from without the shadow.*

3. This loss of light or decrease of intensity at any point outside the shadow is produced by destructive interference. Although interference may result in an increased intensity at some points the integral effect outside the shadow is a loss.

Our main interest is confined to the appearance of the edge of the obstacle causing diffraction.

FIG. 6. *Diffraction Effects at the Filament and Front Aperture.*

A SERIES OF EXPERIMENTS[7]

Fig. 6A is the appearance of an opaque obstacle (filament) when illuminated by parallel light. There appear to be interference bands outside the shadow. These bands are caused by the limited aperture of the observing device.

In Fig. 6B the obstacle has been heated and matched with the incident light. In this case it is seen that the edge of the obstacle is bordered by a dark band invisible in Fig. 6A.

[7] Carried on with the instrument shown in Fig. 1.

Banerji[8] remarks that "one of the most important results indicated (by Sommerfeld's solution of diffraction by a semi-infinite screen) is that the diffraction effect due to the screen may be regarded as due to cylindrical waves emitted by its edge the intensity of which is different in different directions, these waves alone being operative in the region of the shadow, but in the other regions appearing superposed upon and interfering with the reflected and transmitted waves."

Sommerfeld's results as simplified by him[9] are given in the equation

$$s=\cos\left[\frac{2\pi r}{\lambda}\cos(\varphi-\varphi')+nt\right]\mp\cos\left[\frac{2\pi r}{\lambda}\cos(\varphi+\varphi')+nt\right]$$
$$+\frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\left[\pm\frac{1}{\cos\frac{\varphi+\varphi'}{2}}-\frac{1}{\cos\frac{\varphi-\varphi'}{2}}\right]\cos\left(\frac{2\pi r}{\lambda}+\frac{\pi}{4}-nt\right)$$

"where S is the light disturbance, φ' and φ are respectively the angles made by the incident rays and by the radius vector with the plane of the screen, and r is the distance of the point considered from the edge of the screen." The signs depend on the plane of polarization of the incident light.

We will consider the third term which represents the diffracted rays and is, in case the incident light is perpendicular to the obstacle plane

$$\frac{1}{4\pi}\sqrt{\frac{\lambda}{r}}\left[\mp\frac{1}{\cos\theta/2}-\frac{1}{\sin\theta/2}\right]\cos\left(\frac{2\pi r}{\lambda}+\frac{\pi}{4}-nt\right)$$

if θ be measured from the shadow line positive within the shadow. At small angles $\frac{1}{\cos\theta/2}$ is small compared to $\frac{1}{\sin\theta/2}$ and may be neglected. Thus[4] the phase changes sign as we pass the shadow line.

In Fig. 6B, the observing microscope was exactly halved by the shadow line, or rather surface. Thus the light radiating from the edge of the obstacle interfered at the edge of the image of the obstacle to give the dark line. Considering one edge of the obstacle (in this case a flat lamp-filament), light radiating from the edge *into* the shadow, being 180° out of phase with corresponding rays *outside* of the shadow, interfered with the latter to give nearly zero illumination. It is unessential to discuss the fringe system, and we may better pass to the next experiment.

[8] Phil. Mag. *37*, p. 112; 1919.
[9] S. K. Mitra. Phil. Mag. *37*, p. 50; 1919.

In Fig. 6C, the obstacle is illuminated by parallel light as before, but observed through two narrow slits one on each side of and close to the optical axis. The incident light is limited to a narrow beam by a slit on the optical axis. Thus the interference pattern is enhanced in brilliance. None but diffracted rays now enter the telescope. The center of the fringe system (for each edge) is dark. There is nothing new in the appearance of maxima and minima when a line source, the diffracting edge, is viewed through two slits. The first maxima appear in place of the first minima, were the line source a self-luminous source, or an illuminated slit. This interference pattern was produced with white light. It appeared to the observer brilliantly colored, with the center and first maxima very sharply defined. Colors appeared in successive maxima after the first. In Fig. 6D one slit is covered, destroying the interference pattern. That is only light diffracted *into* the shadow at one edge (*out* of the shadow at the other edge) enters the telescope. These two figures illustrate that rays diffracted into and out of the shadow are of equal intensity. Of course it was necessary in taking the photographs to carefully align the telescope, filament, and incident light, in order to obtain the balanced effect.

In Fig. 6E and F, is shown the fringe system when the incident light of the previous arrangement is brought to a focus, so that the diffracting edge is in the image plane. The source of illumination was a tungsten strip perpendicular to the edge. As before none but diffracted rays are visible except at the outer zones of the field. The pattern hardly differs from the previous one.

Now if the two slits before the observing microscope be moved closer or farther apart the ordinary broadening or collapse of the fringe system occurs. However, if placed too far apart the shadow across the image becomes too narrow to distinguish the fringes. This occurs likewise if the single front slit at the objective lens is widened. It can be avoided by using a point source and a rather narrow front slit. If, now, the two slits be gradually widened until the observing telescope is covered by but a narrow opaque strip or wire, the fringes collapse to a fine central minimum bordered by two maxima, exactly as found by Banerji in the case of a diffracting aperture.

Removing the strip or wire permits direct illumination to enter the telescope and more may be learned by again substituting a large source, and heating the obstacle to match the image in brightness. In Fig. 2F, we have the dark line at the edge and a bright line outside of it. In this figure the aperture of the observing device was 0.08 radian and

that of the front objective .03 radian. The bright line or maximum which appears in this arrangement is caused by interference of diffracted rays with incident or unobstructed rays. The absence of more than one maximum is commonly observed when large circular apertures are used. This explanation of the bright line is scarcely adequate and perhaps confusing, but a better explanation will be given later in the discussion of the effect of diffraction by the front aperture.

Before describing the effect of changing apertures in the arrangement of the previous paragraph we will give a brief account of the effect of an obstacle shielding the center of the front aperture. If this be a disc subtending for example 0.06 radian, then the filament or obstacle in the image plane may be viewed with an aperture of say 0.04 radian and appear by diffracted light alone as in Fig. 6G. Each edge of the filament appears as a bright line, and the use of two slits in the observing telescope resolves this into an interference pattern Fig. 6H with a bright center, complementary to the one in Fig. 6E. The symmetry of this arrangement suggests that light diffracted into the shadow and out of the shadow should interfere as before. Then the bright line would be absent. It is a result of the effect of the front obstacle. We may now consider only the light initially diffracted by the disc screening the center of the entrance aperture, i.e. rays which appear to radiate from the edge of this disc. In Fig. 7 light diffracted into the shadow

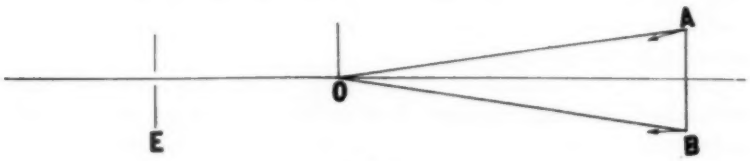

FIG. 7.

at A, reaches the exit orifice E by diffraction into the shadow of O. Light diffracted out of the shadow at B is diffracted out of the shadow of O—into the exit orifice. Thus there is a lack of symmetry and a resulting agreement in phase of the radiation from O at symmetrical points in the orifice E.

By substituting for the obstacle AB, two slits one at A and one at B, and observing O through two slits, all symmetrical about the optical axis, it was possible to form the center of the interference pattern at O with either a dark or bright center. A bright center was produced by covering one of the two slits at A or B, provided careful

adjustment had formed a dark center. A point source was used. But in this case the two front slits were narrow. Light from each converged to O, resulting in a simple interference pattern. The appearance of O depended on whether a maximum or minimum of this pattern fell at O. The slits at A and B could as well have been illuminated by parallel or divergent light except for the intensity of the effect observed at O.

To digress for a few lines, an interesting problem is that of the image of a point source when viewed behind the circular disc AB, through an aperture less than that subtended by AB. The image is very faint with the strongest source, and appears with a dark center. Viewed through a very large aperture it appears similar to that without AB. In place of the expression for the intensity of the image of a point, formed by a circular aperture,[10] $\left(\frac{2}{z}\right)^2 J_1^2(z)$ it can readily be shown that the annular aperture (i.e. with AB) gives for the intensity,

$$\left[\frac{2\pi r_2^2}{z_2} J_1(z_2) - \frac{2\pi r_1^2}{z_1} J_1(z_1)\right]^2$$

where r_2 and r_1 are the outer and inner radii of the aperture.

To return to our main problem: the appearance (or disappearance) of an incandescent filament matched with a larger image in its plane, —consider that diffraction occurs when the wave front is partially obstructed. The image consists of an infinite number of "point images." At points of the image just outside of the edge of the obstacle (filament) there is no obstruction of the wave front. The mere proximity of the obstacle is without appreciable effect. If the image of each point of the source were a point there would be no obstruction whatever. The light of each point image would either pass the edge or not. But not being points, those images at or very near the edge are partially screened from view. The effect depends however *upon the apertures of both objective and observing eyepiece;* that is, upon the direction of the lines of flow of energy through the focus. Suppose for instance that the central rays through the objective followed the geometric rays and converged to a point at the focus. Viewed through a small aperture the image could not be partially obstructed—see Fig. 6 I. But if the image is formed through a small aperture and viewed through a larger one then the image can be partly screened from view as shown in Fig. 6L.

[10] Cf. Gray & Matthews, p. 178.

Chinmayam[11] has recently investigated the flow of energy near a focus, but his work has not yet reached the writers. His treatment of parallel light leads us to conclude that the central rays of light converged by a lens do converge to the central spot of the diffraction ring pattern of the image and that the energy of the rings flows from the diffracting edge of the aperture. This has been proven experimentally by Banerji who focused his eyepiece upon the aperture; and by the writers, in obtaining perfect disappearance of the pyrometer filament. Modification of the Foucault knife-edge test in the following manner further illustrates the fact. Instead of inserting the knife-edge immediately in front of the observing system, the latter is withdrawn along the axis until its aperture is less than that of the front objective—See Fig. 8. It is still focused upon the objective. If the knife edge be inserted to barely screen the central spot of the image of a point source, the field will appear quite perfectly black. Slightest lateral motion of the knife edge illuminates or darkens the field suddenly, and if the exit aperture is rather small a poor front lens appears to furnish a very good image, that is the change takes place very suddenly.

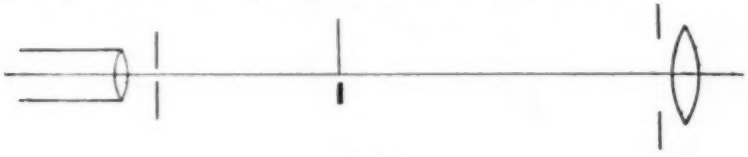

FIG. 8.

Likewise, when the pyrometer filament is viewed with a small aperture and matched with an image formed by a much larger one, the images of points of the source are effectively points, not partially obstructed as in Fig. 6L, but passing the edge in full brightness as shown in Fig. 6I though very close to the edge. If the exit aperture be increased to nearer that of the entrance aperture a dark line appears at the edge of the filament. In this case rays just grazing the edge are *partially obstructed*. The bright line of Fig. 2F just outside the dark line does not appear until the exit aperture is sensibly larger than the entrance, *when the diffraction ring patterns*—the images of points—*some distance from the edge are partially obstructed*.

We have repeated Banerji's experiments focusing the eyepiece upon the entrance aperture, using however, the arrangement of Fig. 8.

[11] Indian Assoc. for Cultivation of Science 7; 1921.
See also Phil. Mag. *37*, p. 9; 1919.

If the entrance aperture is smaller than that of the observing eyepiece, it appears by diffracted light as shown in Fig. 6J, similar to an illustration of Banerji's experiments. With our arrangement the exit aperture can be made smaller than the aperture viewed, and seen thus by diffracted light it appears as in Fig. 6K. But one bright line appears, that is, only light diffracted into the shadow of the front aperture enters the observing telescope. Fig. 6K was obtained with a two hour exposure. When the entrance aperture is gradually increased from less to greater than the exit, its edge appears strongly illuminated at first, suddenly diminished at equality and grows rapidly dimmer when greater than the exit. Thus in Fig. 2F the diffracted rays from the front aperture are strongly effective, while in Fig. 2D, but slightly effective, and in Fig. 2B, the effect is negligible,—*light diffracted by the front aperture illuminates the exit aperture very feebly.*

A NEW LAW OF DIFFRACTION

The statement has been made above that the visible effect of diffraction by an obstacle in an image plane disappears when the diffracted rays radiating from the edge of the entrance aperture illuminate the exit but feebly, that is the diffraction by the obstacle in this case results from initial diffraction by the aperture. We may consider the incident light as made of two components, one undistorted and following the geometric rays and one as of light radiating from the edge.

Banerji[4] has suggested the mathematical problem of thus expressing the light disturbance for circular apertures as a surface integral and a line integral for the edge. This line integral would furnish a means for expressing the decrease in illumination of the exit aperture by diffracted light from the entrance,—such decrease with increase of the difference in entrance and exit. Without this expression we have used one representing the rate of decrease in size of the diffraction image with increase in aperture, which is the same thing.

Experiment alone, in any case, could reveal what magnitude of aperture would produce an invisible effect of diffraction.

We may form a law of diffraction as:

"Of light incident on an obstacle in an image plane, only that component is diffracted, which represents the diffracted rays radiating from the edge of the aperture."

Is diffraction in a focal plane a special case? In the diverging cone of light, diffracted rays radiating from the edge of an obstacle in the

focal plane likewise are diverging. The diffracted rays are coherent with direct rays. Moreover for every ray diffracted into the shadow of the edge, there is a ray following the same path which has been diffracted out of the shadow of a different portion of the converging rays. These two rays will be just 180° out of phase and give no resultant effect. That is within the central portions of the diverging cone of light there is no resultant effect of diffraction. Such a descriptive explanation as this is not necessary, and only answers a false conception of diffraction: that light is diffracted merely as a result of its proximity to the edge in passing. The only effect of such proximity is that of Einstein's theory of gravitation.

The resolving power of the microscope has been until recently derived differently than that of the telescope sighted upon self-luminous objects, but A. W. Porter[12] points out that the derivation should be the same when the illumination is such that every point of the image formed by the substage condenser acts as an independent source. Porter states that with random illumination "each point of the source will send light practically in one phase to a large patch of the object." His explanation of why this reduces resolving power is that two dependent or coherent radiating points spaced at the limit of resolution (for self-luminous objects) will appear as one because of the interference of the rays from each. This is less direct than to refer to whether or not the object acts as an obstruction in the path of the rays.

There is of course a marked similarity in the problem of illumination in microscopy, and in optical pyrometry. For example, Porter states in the same article that "much depends upon the character of the light transmitted through the object when it is semi-transparent or *reflected* from it when it is opaque."

The optical pyrometer with a round filament has plainly shown a part which is played by reflection. But further than this our study has revealed why in the use of a microscope the greatest resolving power is not attained until the condenser aperture exceeds that of the microscope objective. On the other hand the greatest resolving power has been found in some cases to result when a particular value of the condenser aperture is used. Such an effect is closely associated with our results for the round filament. Microscopic particles presenting an edge effect of reflection or absorption will modify the proper aperture of the condenser necessary to afford the greatest resolving power.

[12] Trans. Far. Soc. *16*, 1, p. 67; 1920.
Lord Rayleigh made the same point in Phil. Mag. *42*, 1896.

We are not particularly concerned about the relation of our problem to that of microscopy, and our unfamiliarity in that field may have resulted in our repeating work which has been done. However, a careful search of the literature has assured us that we have not repeated anything that has been recorded.

Our reference to the field of microscopy is incomplete without mentioning that we have striven for *perfect disappearance* of the object (filament) while the microscopist has desired to attain *greatest visibility* of the object.

At what relative value of apertures can be expected a negligible effect of diffraction? Suppose the exit aperture is negligibly small. Then the change in the effect with varying entrance aperture can be represented by the rate of shrinkage of the diffraction image of a point source by an aplanatic lens. Lommel's results for a circular orifice give the intensity of illumination at any point, P, in the image plane

$$I = \frac{1}{\pi\rho^2} \; J_1^2 \left(\frac{2\pi\rho\theta_1}{\lambda} \right) \tag{1}$$

where ρ is the distance of P from the optical axis and θ_1 is the half angular aperture. I is taken as unity at the aperture and πr^2 is put equal to unity, r being the radius of the aperture.

Lord Rayleigh has considered the total illumination within any value of $z = \frac{2\pi\rho\theta_1}{\lambda}$, or, when $J_1(z) = 0$, within any dark ring of the image pattern. Whatever the value of θ_1 the total illumination within any dark ring remains constant. But the total illumination within any fixed area of diameter ρ_1 changes with aperture, and is

$$I_T = \int_0^{\rho_1} 2\pi\rho I d\rho = 2 \int_0^{z_1} \frac{1}{\rho} J_1^2(z) d\rho$$

$$= 2 \int_0^{z_1} \frac{1}{z} J_1^2(z) dz \text{ where } z = z_1 \text{ when } \rho = \rho_1 \tag{2}$$

$$= 1 - J_0^2(z_1) - J_1^2(z_1) \text{ where } z_1 = \frac{2\pi\rho_1\theta_1}{\lambda}$$

The rate of shrinkage of the ring pattern with increasing θ_1 is found from

$$\frac{\delta}{\delta\theta_1} I_T = \frac{\delta}{\delta\theta_1} \left(-J_0^2(z_1) - J_1^2(z_1) \right) = \frac{2}{\theta_1} J_1^2(z_1) \tag{3}$$

since $\frac{d}{dx} J_0 = -J_1$ and $\frac{d}{dx} J_1 = J_0 - \frac{1}{x} \cdot J_1$

The values of $\frac{\delta}{\delta\theta_1} I_T$ for $\rho_1 = 0.006$ mm and 0.024 mm are given in Table 3 and shown graphically in Fig. 9.

TABLE 3

$\rho_1 = 0.006$ mm		$\rho_1 = .024$ mm	
θ_1—radians	$.953 \times \frac{\delta}{\delta\theta} I_T$	θ_1—radians	$.953 \times \frac{\delta}{\delta\theta} I_T$
.003	5.104	.001	27.05
.005	8.386	.002	51.80
.010	15.67	.004	86.77
.01167	17.68	.00567	96.00
.020	23.66	.008	80.31
.02267	24.00	.010	51.58
.030	21.48	.012	23.23
.040	12.89	.01596	0
.050	4.382		
.06386	0		

If we assume that to avoid visible diffraction (at the filament) θ_1 should be increased until within some arbitrary value of ρ_1 the rate of increase has reached its first minimum, then $J_1(z_1) = 0$ and ρ_1 is the radius of the first dark ring. If $\lambda = 2\pi \cdot 10^{-4}$mm, $\rho_1\theta_1 = 0.000383$.

If we select the first maximum of $\frac{\delta}{\delta\theta_1} I_T$, then $\frac{\delta^2}{\delta\theta_1^2} I_T = 0$, or $\frac{\delta^2}{\delta\theta_1^2} I_T =$

$$\frac{\delta}{\delta\theta_1}\left(\frac{2}{\theta_1} J_1^2(z_1)\right) = \frac{4}{\theta_1^2} J_1(z_1)\left[2z_1 J_0(z_1) - 3J_1(z_1)\right] = 0 \qquad (4)$$

Then

$$2z_1 J_0(z_1) = 3J_1(z_1)$$

or $\frac{J_0(z_1)}{J_1(z_1)} = \frac{2}{3} z_1$. An approximate solution is easily found by scanning parallel tables of J_1 and J_0 giving

$z_1 = 1.36$ or $\rho_1\theta_1 = 0.000136$ if $\lambda = 2\pi \cdot 10^{-4}$

Such a value of $\rho_1\theta_1$ corresponds to a ρ_1 which includes practically the entire central spot. If the exit aperture is very much smaller than the entrance, only a fraction of the energy entering the area of the central spot enters the exit aperture. Lord Rayleigh found that the

intensity I of equation (1) is constant for some distance from the axis. Its value drops by about 10% below that on the axis when $\rho_1\theta_1 = 0.00007$.

The values of ρ_1 and θ_1 when $\rho_1\theta_1 = 0.000383$, 0.000136, and 0.00007 are given in Table 4 and shown graphically in Fig. 9. These curves show that the diffraction ring pattern is very large when the aperture is a pinhole and as the aperture increases in size the central spot of the image shrinks very rapidly until an aperture of a few hundredths of a

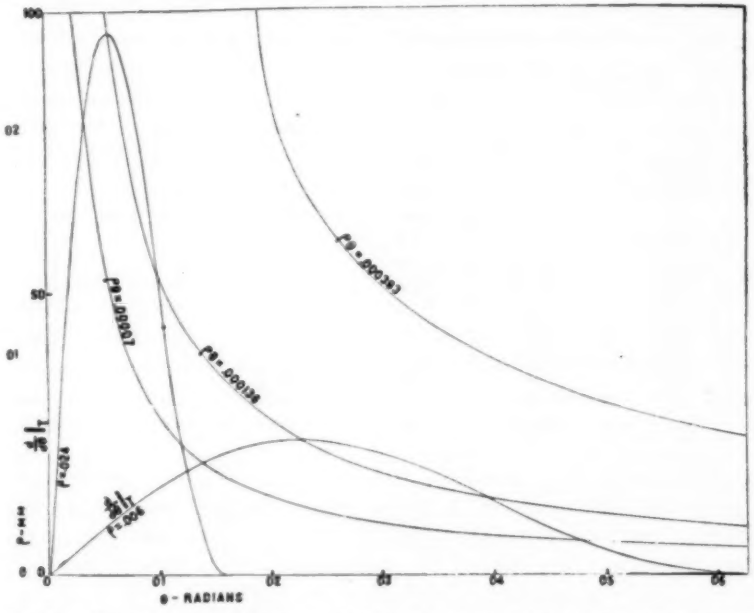

FIG. 9. *Change of Integrated Intensity of Diffraction Image of Point Source with Change of Aperture.*

radian is reached, then more slowly, and beyond 0.10 radian the pattern shrinks but slightly. We can expect a visible diffraction effect until the main portion of the central spot, i.e., within $\rho_1\theta_1 = 0.000136$, shrinks to a few wave lengths, corresponding to 0.02 or 0.03 radian for θ_1. This magnitude of θ_1 is necessary for *a negligible illumination of a minute exit aperture by diffracted rays radiating from the edge of the entrance aperture.* The effect is angular and the magnitude found becomes the difference between exit and entrance apertures. Referring again to the curves of $\rho_1\theta_1$, as the apertures increase in size the differ-

ence must increase not only because of the slow shrinking at large apertures, but also because the resolving power of the observing eyepiece increases, and a smaller and smaller effect is visible. When the exit aperture is 0.06 radian or more, the dark line at the edge of the obstacle if present, appears sharply defined. It is greatly softened and broadened with an exit of 0.02 radian and at 0.005 radian, two edges of a 2 mil filament are not resolved—the dark lines are invisible whatever entrance aperture is used.

TABLE 4

$\rho_1\theta_1=0.00007$		$\rho_1\theta_1=0.000136$		$\rho_1\theta_1=0.00038317$	
ρ_1(mm)	θ_1(radians)	ρ_1(mm)	θ_1(radians)	ρ_1(mm)	θ_1(radians)
0.0005	0.14	0.0005	0.272	0.0005	.....
.001	.07	.001	.136	.001	.....
.002	.035	.002	.068	.002	0.1916
.003	.023	.003	.0453	.003	.1237
.004	.0175	.004	.034	.004	.09575
.006	.0117	.006	.0227	.006	.06383
.008	.00875	.008	.017	.008	.047875
.010	.007	.010	.0136	.010	.038317
.015	.00467	.015	.00907	.015	.02554
.020	.0035	.020	.0068	.020	.01916
.024	.00292	.024	.00567	.024	.01597
.040	.00175	.040	.0034	.040	.00958

The experimental values for the necessary difference between entrance and exit apertures to obtain perfect disappearance of the flat filament are so nearly those which we have just concluded are necessary, that doubt remains of the significance of lens imperfection. If it is recalled that diffraction by the filament is wholly a result of diffraction by the entrance aperture, or in other words, is a result of partial obstruction of light radiating from the edge of the front aperture, then lens imperfections play a minor part. We have found that various lenses of wide differences in correction for spherical aberration give sensibly the same results in aperture tests. Whether all the lenses used were so far from perfect that their differences were too slight to reveal an effect remains to be proven. The optical division of the Bureau of Standards has undertaken to compute a lens for least spherical aberration. It is readily apparent that corrections for color or coma are unnecessary, or at least secondary. What is essential, is that a "point image" be as small as possible. A filament used in practice is never

more than 0.10 mm in diameter and the observer focuses attention directly upon it. Hence flatness of the image is unnecessary. It is desirable to obtain a fairly well formed field of observation to aid the observer in focussing the eye, and to promote a favorable psychological effect.

Suppose the pyrometer is sighted into an experimental black-body. The field is uniformly illuminated; hence there are no practical effects of lens imperfection. Any plane within the black-body can properly be considered the plane of the source. To account for the absence of any effect of spherical aberration it is only necessary to consider a new plane for the source, for each zone of the lens. Experimentally the problem is simple. It is found that when the source is large, the filament can be made to disappear, when the image of the source is somewhat removed from the plane of the filament. Thus lens imperfection requires attention only when sighting upon a small source such as a minute experimental black-body opening or a lamp filament.

During our experiments, we observed at one time that with certain large apertures a round filament of a pyrometer lamp appeared with bright edges when matched with a large image, but disappeared perfectly when matched with a rather small image. The bright edges were caused by reflection of more oblique rays from the large source. Because of spherical aberration of considerable magnitude, the more oblique rays from the small source did not fall upon the pyrometer filament. In this connection it is well to remark that the best apparent focus of the source sighted upon with the pyrometer, is not the best focus for the front objective if the exit aperture stop is removed. If the whole objective is used in sighting upon a small source, we have noticed that disappearance is obtained, if the source is small only when the source is apparently slightly out of focus. Of course, this effect is only observed when the lens is used in a position to barely fill the required entrance aperture. By temporarily removing the exit stop, then focussing upon the source, and replacing the stop, it was found naturally that the focus thus obtained was the same as that found by adjustment to disappearance of the pyrometer filament.

The independent effect of the exit aperture in forming an imperfect image is secondary.

Suppose the glowing filament is viewed upon a dark back-ground. With a small aperture, two interference maxima will appear, one on each side of the filament. They are quite well defined if a slit is used, but blurred by a circular aperture. The filament may appear with

one or more minima within its edges. If the filament is now darkened and viewed upon a bright background a complementary pattern results. There can be no question that when the filament matches the background, such effects are obliterated.

Sectored discs are used in optical pyrometry for measuring temperatures above the upper safe limit of the pyrometer filament, and are sometimes used in calibrating the pyrometer. Is the transmission of a sector disc as determined mechanically with a circular dividing engine the correct value to use in pyrometry? Certainly if a sector has very narrow openings and is so placed that the opening subtends a small angle at the source or image, a large fraction of the transmitted light, is diffracted. The diffraction image of a point source is in this case greatly elongated in a series of maxima and minima.

If the narrow opening of a sector be placed across the center of the objective lens of the pyrometer, a point source appears as a well defined diffraction pattern. Rotating the disk blurs this image because the lens is not perfect. If the sector opening is comparable with the diameter of the lens, definition is not appreciably effected. Or the disc may be placed sufficiently near the source or image so that the aperture corresponding to the opening is as large as necessary. If the image is small then a decrease in definition may decrease the brightness even to the center. Unless the source is large in at least one dimension—when the sector opening can be placed perpendicular to the larger dimension—the disc of low transmission cannot be used safely.

If the source is large or the opening of the sector is large, the disc may be used in any position.

Another cause for uncertainty in the use of a sector disc with a small opening, when sighting upon small sources, is the combined effect of spherical aberration and diffraction. The interposition of the sector disc with the pyrometer does not alter the values of entrance and exit apertures required to obtain disappearance of the filament. With such apertures the rotating sector performs a mechanical intermittent integration across the cone of rays, such that there is as much light diffracted *into* the exit when the opening covers outer zones of the cone of light as is lost by diffraction *out of* the exit. In other words, the diffracted rays from the edge of the sector opening radiate uniformly although intermittently across the whole entrance aperture. Spherical aberration will prevent this integration if the source is too small, so that while disappearance of the filament may occur without

the sector disc, it will not, with a sector disc of small openings. However, in case the objective of the pyrometer is a wide aperture lens, and is so placed that the entrance aperture utilizes only a part of it (magnification of the source is low) the surprising fact appears that definition of the source may be much decreased by a sector disc, while still no effect of diffraction by the filament is visible!

Sectored discs of low transmission should be made with only one opening in order (1) to affect definition the least and (2) to obtain the highest accuracy in measuring the transmission with a circular dividing engine. The diameter of the disc should be as large as convenient so that the opening for a given transmission is as large as possible. For example a disc 20 cms in diameter, transmission 0.002 will have an opening 1.2 mm in width near the circumference. This disc may be used with good results 5 or 6 cms from the source or image. The use of a sector smaller than this is inadvisable, and combination with an absorbing screen or glass should be used for extrapolating over a greater range of temperature than this affords. A transmission of 0.002 corresponds approximately to an extrapolation (for red light) from 1000° to 1700°C or from the melting point of palladium (1555°C) to the melting point of tungsten (3400°C).

A test for the presence of a diffracted component of light in the diverging cone may be applied by measuring, if possible, the partial polarization taking place. However, the plane of polarization of diffraction is parallel to that of radiation from the sides of the filament. The light from the filament is polarized to a considerable extent, while at small angles of diffraction polarization is negligible. Such a test will probably fail. Worthing and Forsythe attempted this and evidently found an easily measured effect, which probably was the result of conditions not apparent in their published work. They concluded from this result that the effect at the filament was mainly one of diffraction *into* the shadow because Gouy and Wien have found that light diffracted into the shadow is partially polarized. However Sommerfeld[13] shows that the light diffracted outside of the shadow is likewise polarized and to the same extent (excluding direct rays). Experimental verification of this fact has not so far as the writers know, been heretofore recorded. It was observed in the following manner.

As stated above polarization at small angles of diffraction is negligible. But at a large angle light reflected by the filament is appreciable,

[13] Mitra, S. K., loc. cit.

so in order to eliminate this complication, a sharp edged slit (razor edges) was substituted for the filament. The direct illumination of the exit aperture was cut off by screening all of the lens but a narrow segment so that the bright bands of diffracted light from the two edges of the slit would be visible inside the shadow of one and outside the shadow of the other. See Fig. 10. Observation of the bands through a Nicol prism did not destroy their equality of brightness, whatever angle of the Nicol was used. Polarization of the incident light perpendicular to the slit gave barely visible inequality of brightness of the two bands (with or without Nicol analyser). Polarization of the incident light parallel to the slit gave equality of brightness. Thus the effect of appreciable thickness of the razor edges is apparent, destroying exact symmetry of the diffracted rays about the shadow line. None of these effects are noticeable if the diffraction angle is small (.05 or .06 radian).

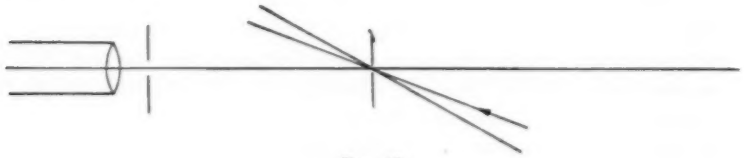

FIG. 10.

Substituting the pyrometer filament (unheated), reflection occurs. Light diffracted out of the shadow is apparently greater because of the added reflected light. The latter is largely polarized perpendicular to the edge. Observation through an analyser set to exclude reflected light equalizes the two bands if the filament is small (2 mils). The light reflected by an 8 mil filament is not sufficiently polarized to permit this. Polarization of the incident light parallel and perpendicular to the filament, with observation through a Nicol crossed with the polarizing prism, augments somewhat the differences caused by the polarization of diffracted and reflected light, which are polarized at right angles. As above none of the effects are noticeable if the incident light is not very oblique, i.e. if the diffracted light is observed from a small angle with the shadow line.

The light reflected by a small filament is almost negligible. That is the small filament will disappear in matching, with a large entrance angle. Shielding direct rays permits the eye to distinguish light of intensity equal to a very small fraction of a per cent of the incident intensity. The few experiments just described, were carried out with

the pyrometer sighted on a tungsten strip lamp operated at about 2300°C. A strip lamp was used to obviate the troubles caused by spherical aberration. The image of the strip was placed perpendicular to the diffracting edges of filament and slit, giving thus the effect of a very large source of great intensity.

Shielding the incident rays can be accomplished by crossing the polarizer and analyser, to observe perhaps diffracted rays from the filament without otherwise shielding direct rays. However, the diffracted light is always polarized in the same plane as the incident light, as first shown by Gouy, at least when the plane of polarization is parallel or perpendicular to the edge.

WASHINGTON,
FEB., 1923.

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A Photoelectric Method for Testing the Wien-Planck Radiation Law in the Ultraviolet and Visible Range.—To test the Planck black body formula in the blue and ultraviolet regions (where it becomes practically identical with the Wien formula) it is necessary to use a more intense radiator and more sensitive intensity-measuring instrument than the heated cavity and the thermal receivers used in work in the infrared have yet been made. Steinke used as radiator a carbon lamp-filament, which, according to a series of researches in Lummer's laboratory, emits radiation like that of a black body except for a factor independent of λ and T, not essential for the test; a way of determining its temperature from its heating-current has also been worked out; the temperature-range used was about 1480°–2170° K. To separate out nearly monochromatic light, Steinke used (unfortunately) colored filters; the transmitted beams were considered equivalent to monochromatic light of wave lengths 316, 340, 462, 535, 630mμ respectively. An argon-filled potassium-lined photoelectric cell of the Elster and Geitel type, made of Uviol glass, served to measure the intensity of the light. Much space is devoted to the behavior of this cell, notably its fatigue-phenomena and its current versus illumination (i versus L) relation; i was found to increase as L^z, where z is greater than unity and varies with wave length and cell-voltage, and had to be determined anew for each run. Steinke verified that log L was a linear function of $1/T$, as it should be by Wien's law:

$$L = \text{const.} \times e^{-c_2/\lambda T}$$

and for c_2 he obtains from the slopes of the five lines (the isochromatics) the values 14458, 14380, 14302, 14344, 14476, in good agreement with previous accurate work at longer wave lengths. (The other constant involves the factor aforesaid, characterizing the carbon filament). [E. Steinke, ZS. f. Physik, *11*, pp. 215–238; 1922.]

KARL K. DARROW

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